Polyhedral Scheduling in the R-Stream Compiler

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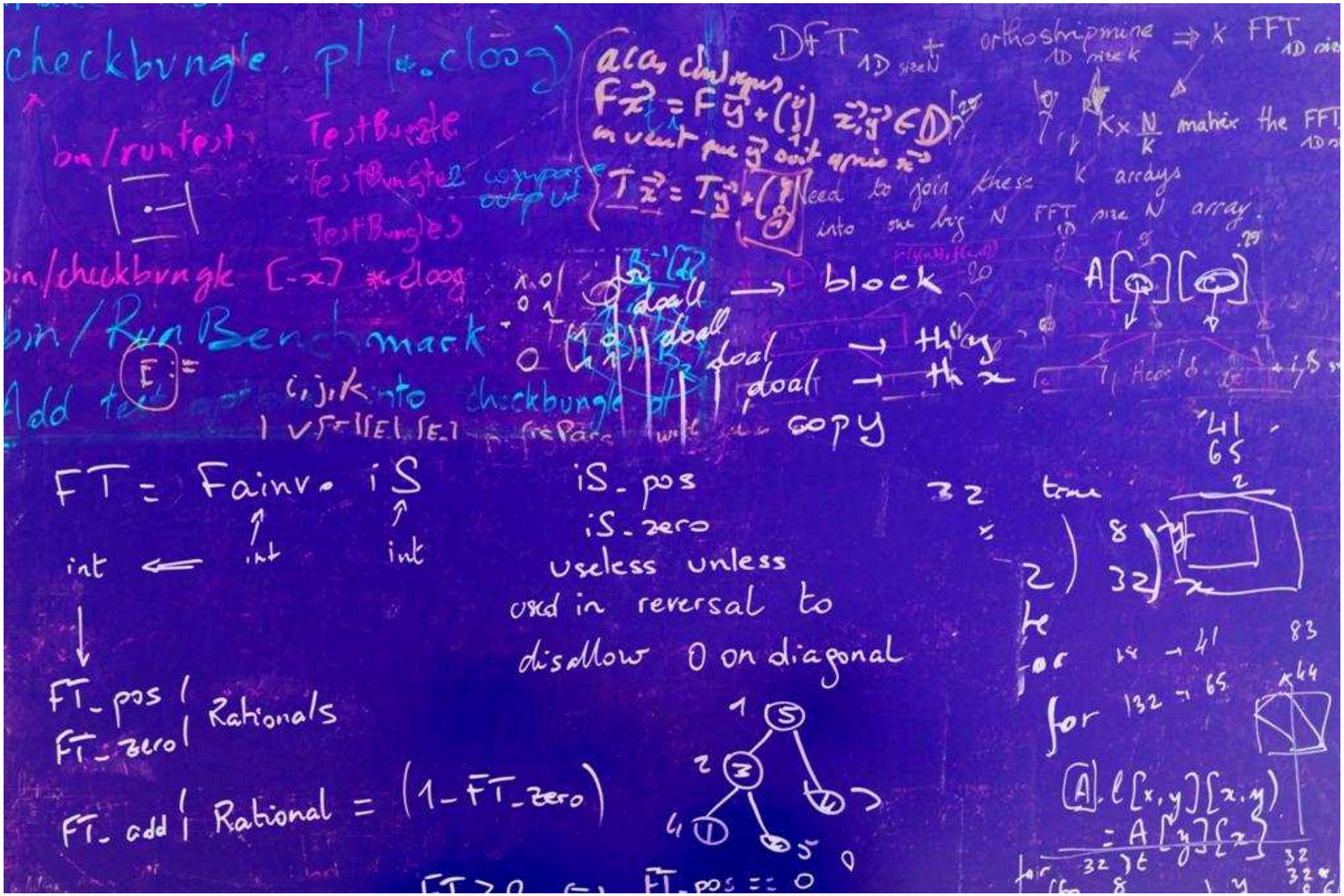


Outline

R-Stream Overview

Balancing Parallelism and Memory

Joint Vectorization and Data Layout Formulations

Results

Conclusions

Benefits of Automatic Parallelization

Optimizations automatically achieved

- Programmer writes at very high level
- Instead of hand coding

Ability to quickly generate code (with these optimizations)

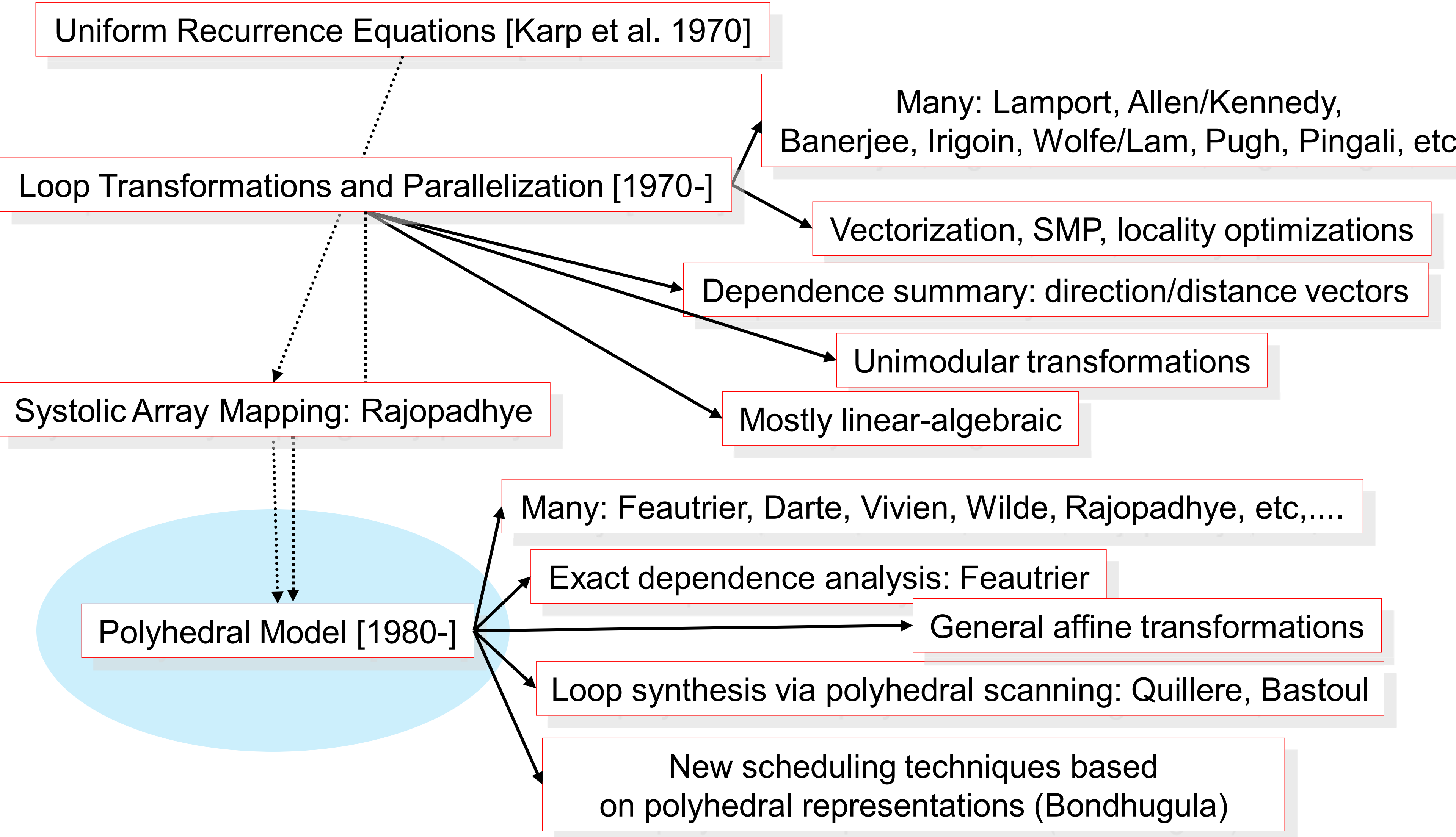
- Substantial coding effort if done manually

New optimizations targeted at future architectures

- Parallelism – locality - other tradeoffs (energy)
- Explicit communication management
- Deep hierarchy (on-going)

Ability to automatically generate various code variants with tunable parameters

Enabling technology is polyhedral abstraction



Polyhedral model – challenges in building a compiler

Mathematical abstraction is not trivial

Scalability of optimizations / representation / code generation

Traditionally confined to dependence preserving transformations

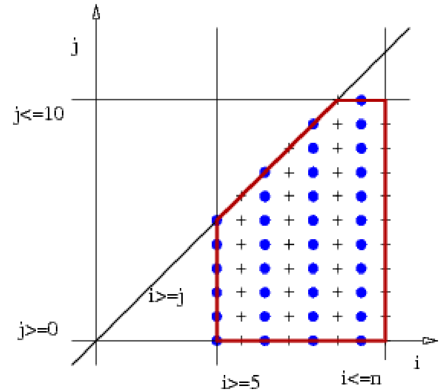
Code can be radically transformed – outputs can look wildly different

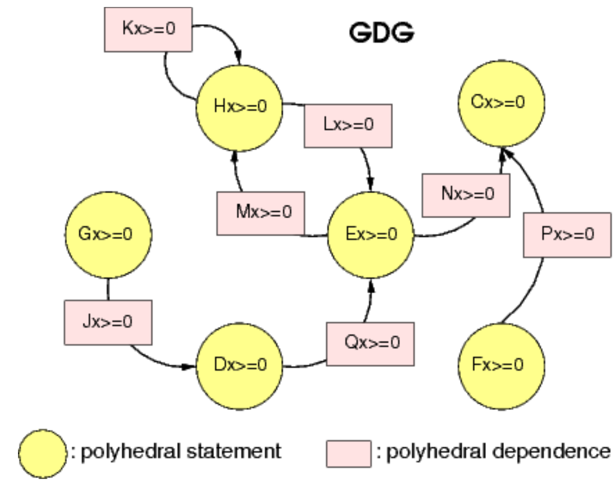
Modeling indirections, pointers, non-affine code.

Some of these challenges are solved + other on-going ideas

R-Stream model: polyhedra model

```
n = f();
for (i=5; i<= n; i+=2) {
  A[i][i] = A[i][i]/B[i];
  for (j=0; j<=i; j++) {
    if (j<=10) {
      … A[i+2j+n][i+3]…
    }
}
```





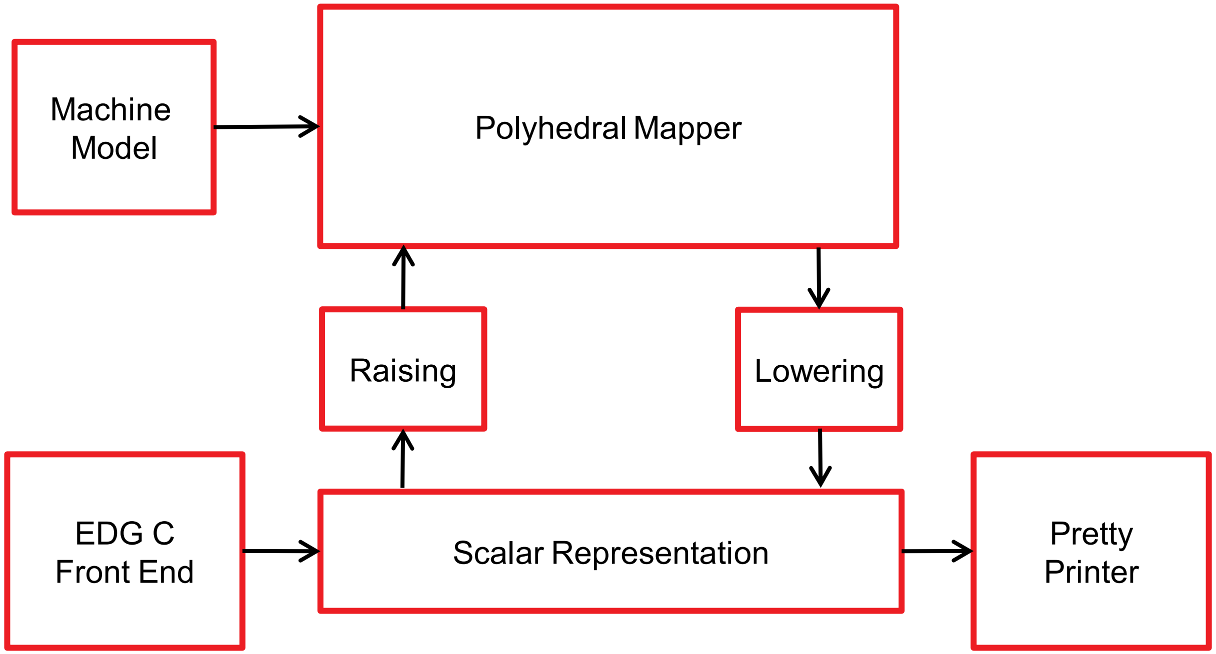
$$\{i, j \in Z^2 \mid \exists k \in Z, 5 \le i \le n; 0 \le j \le i; j \le i; i = 2k+1\}$$

$$\begin{pmatrix} A_0 \\ A_1 \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ j \\ n \\ 1 \end{bmatrix}$$

Affine and non-affine transformations
Order and place of operations and data

Loop code represented (exactly *or conservatively*) with polyhedrons
→ High-level, mathematical view of a mapping
→ But targets concrete properties: parallelism, locality, memory footprint

R-Stream blueprint



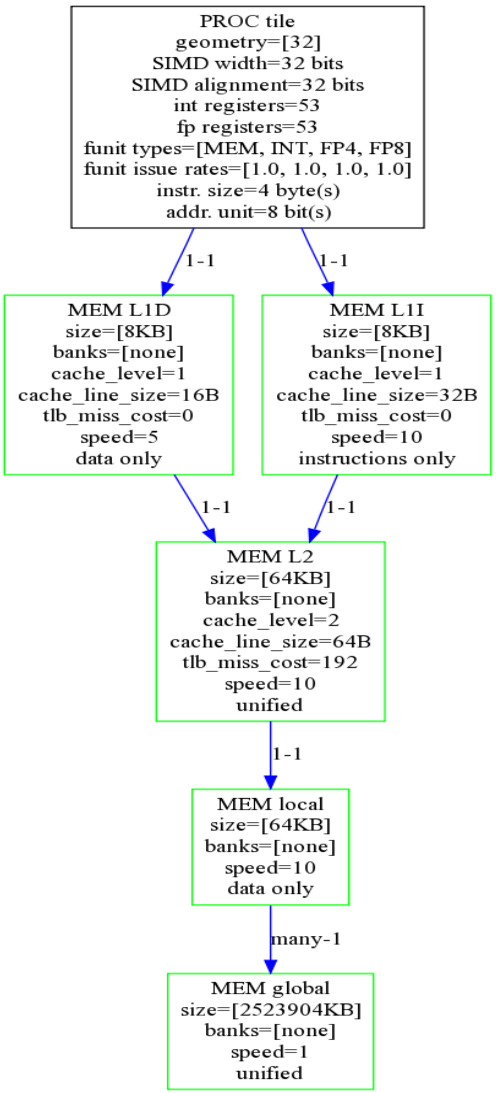
Driving the machine mapping: the machine model

Target machine characteristics that have an influence on how the mapping should be done

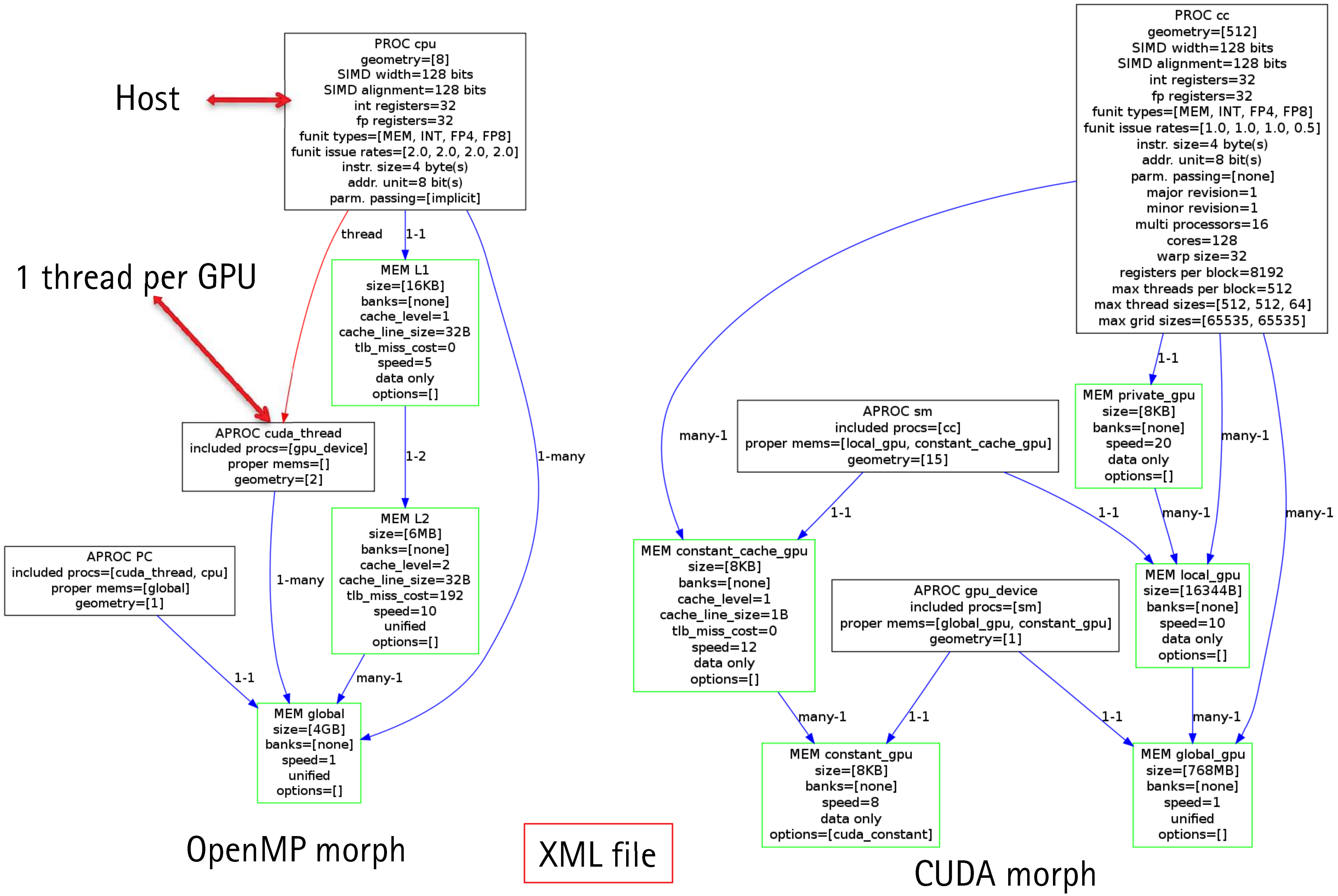
- Local memory / cache sizes
- Communication facilities: DMA, cache(s)
- Synchronization capabilities
- Symmetrical or not
- SIMD width
- Bandwidths

Currently: two-level model (Host and Accelerators)

XML schema and graphical rendering

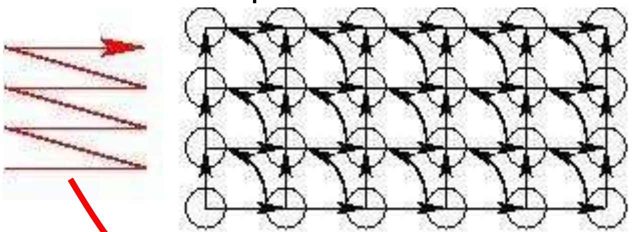


Machine model example: multi-Tesla

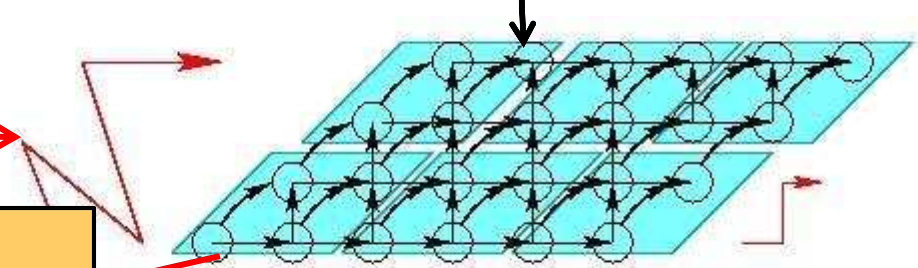


Mapping process

Dependencies

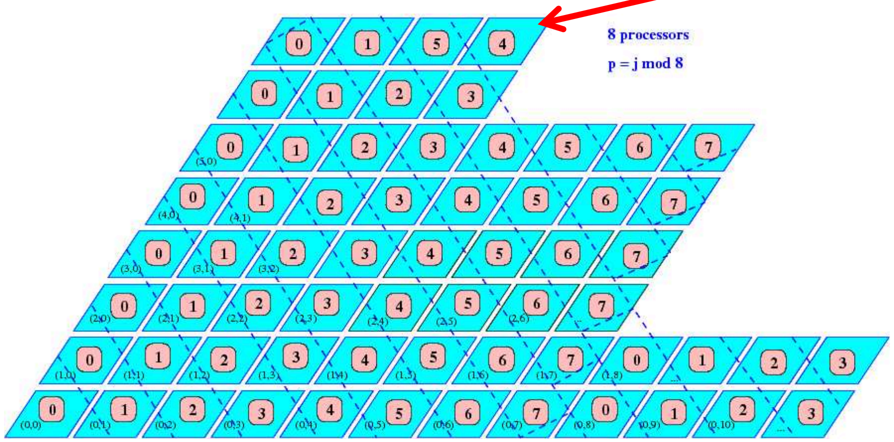


2- Task formation:
- Coarse-grain atomic tasks
- Master/slave side operations



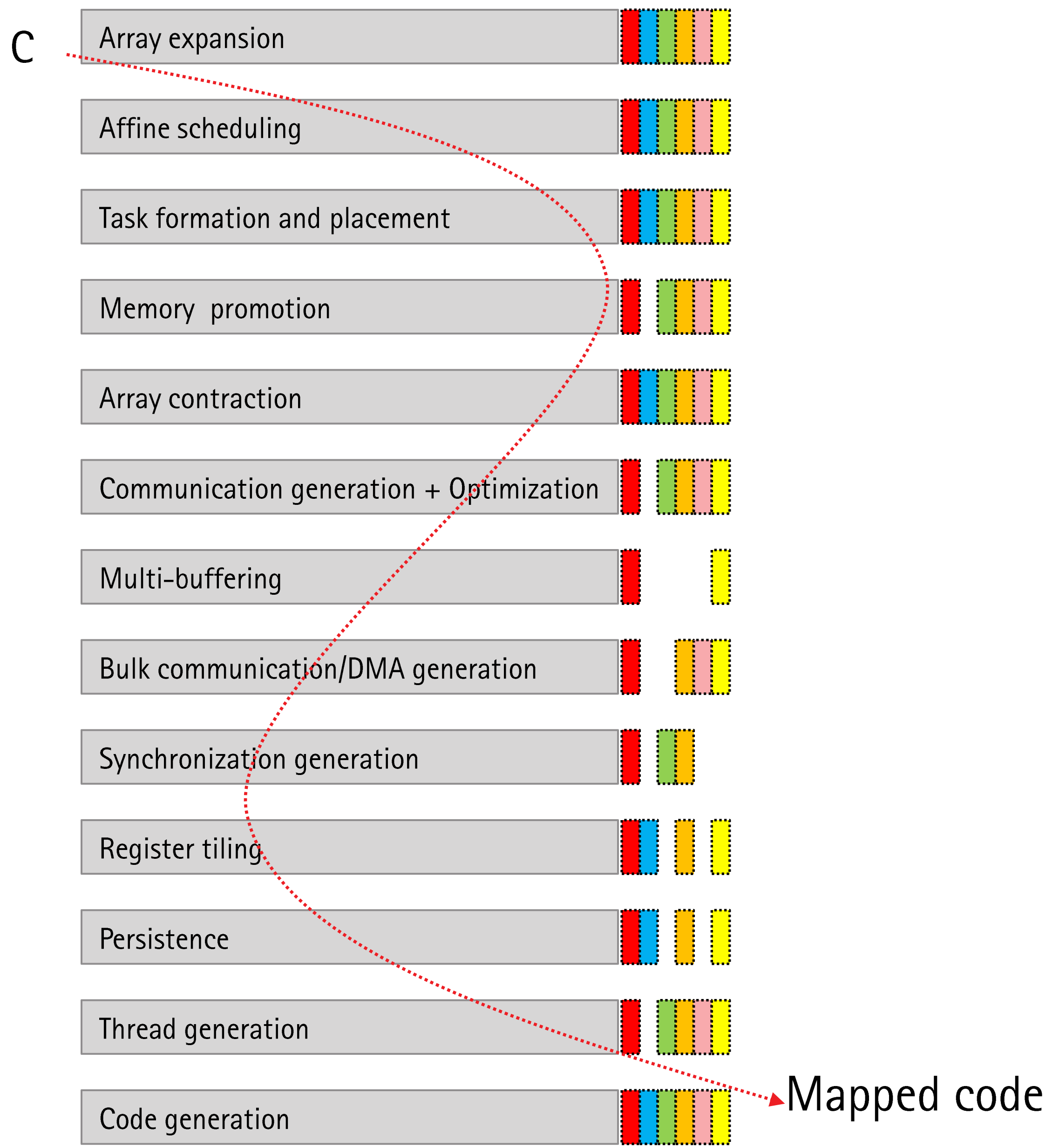
1- Scheduling:
Parallelism, locality, tilability

3- Placement:
Assign tasks to blocks/threads

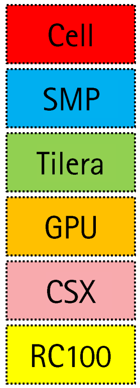


- Local / global data layout optimization
- Multi-buffering (explicitly managed)
- Synchronization (barriers)
- Bulk communications
- Thread generation -> master/slave
- CUDA-specific optimizations

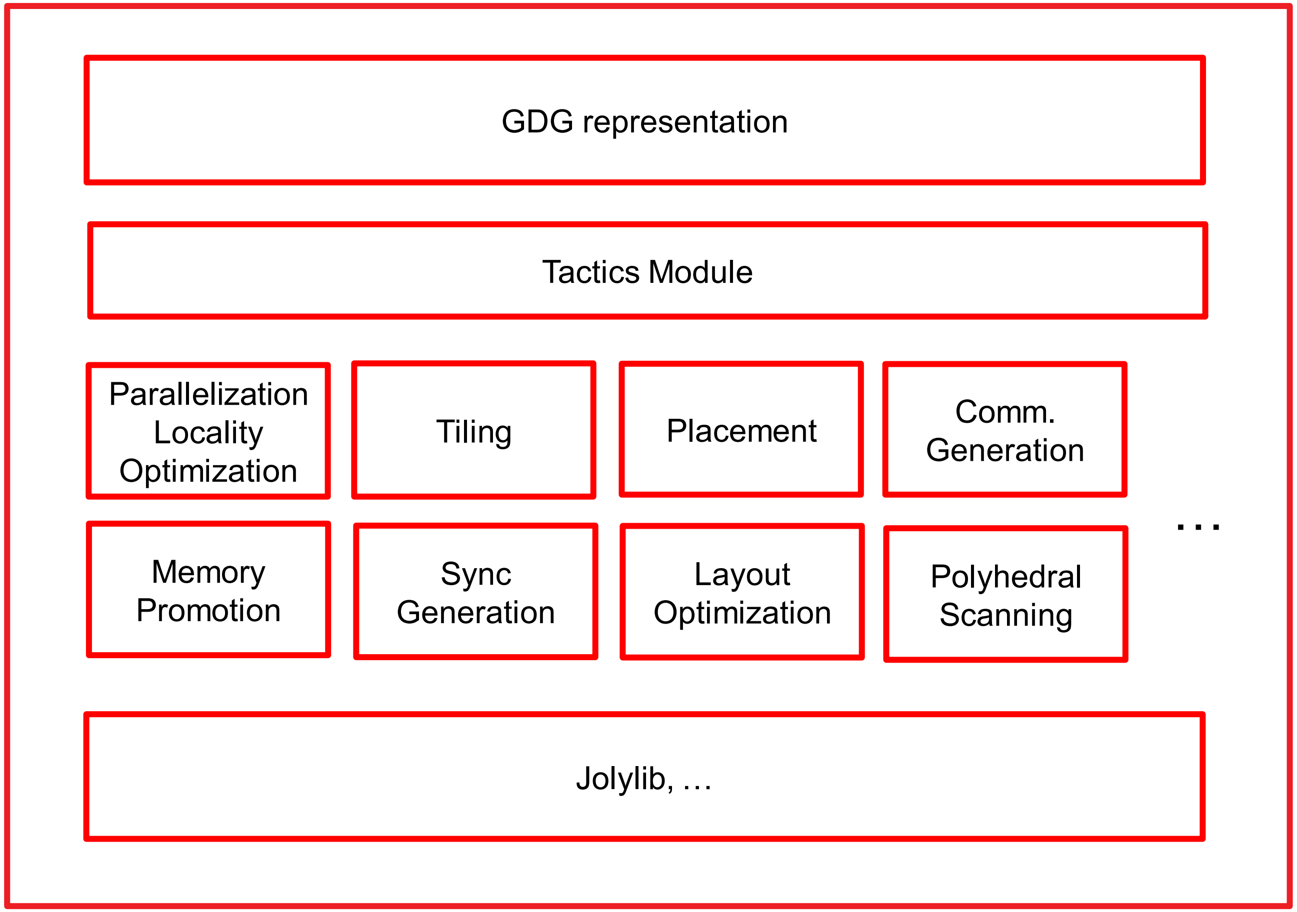
Mapper flow



Significant reuse of modules across targets

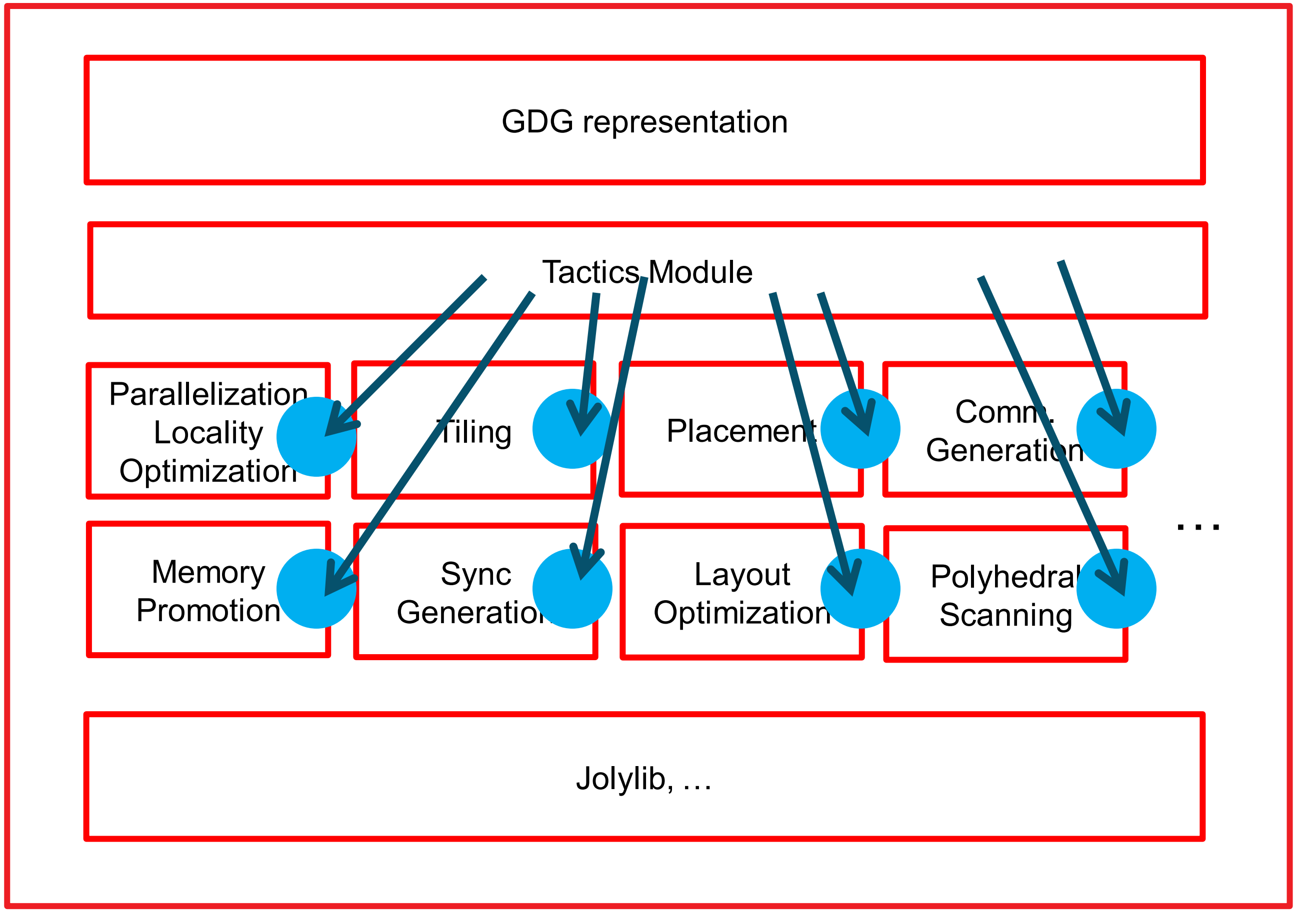


Inside the polyhedral mapper



Inside the polyhedral mapper

Optimization modules engineered to expose “knobs” that could be used by auto-tuner



Loop transformations (URUK-based representation)

```
for(i=0; i<N; i++)
  for(j=0; j<N; j++)
    S(i,j);
```

unimodular

permutation

```
for(j=0; j<N; j++)
  for(i=0; i<N; i++)
    S(i,j);
```

$$\theta(i,j) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix}$$

reversal

```
for(i=N-1; i>=0; i--)
  for(j=0; j<N; j++)
    S(j,i);
```

$$\theta(i,j) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix}$$

skewing

```
for(i=0; i<N; i++)
  for(j=α*i; j<N+α*i; j++)
    S(i,j-α*i);
```

$$\theta(i,j) = \begin{bmatrix} 1 & 0 \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix}$$

scaling

```
for(i=0; i<α*N; i+=α)
  for(j=0; j<N; j++)
    S(i/α,j);
```

$$\theta(i,j) = \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix}$$

Loop fusion and distribution (URUK-based representation)

```
for(i=0; i<N; i++)
  for(j=0; j<N; j++)
    S1(i,j);
  for(j=0; j<N; j++)
    S2(i,j)
```

$\xrightarrow{\text{fusion}}$

$\xleftarrow[\text{distribution}]{}$

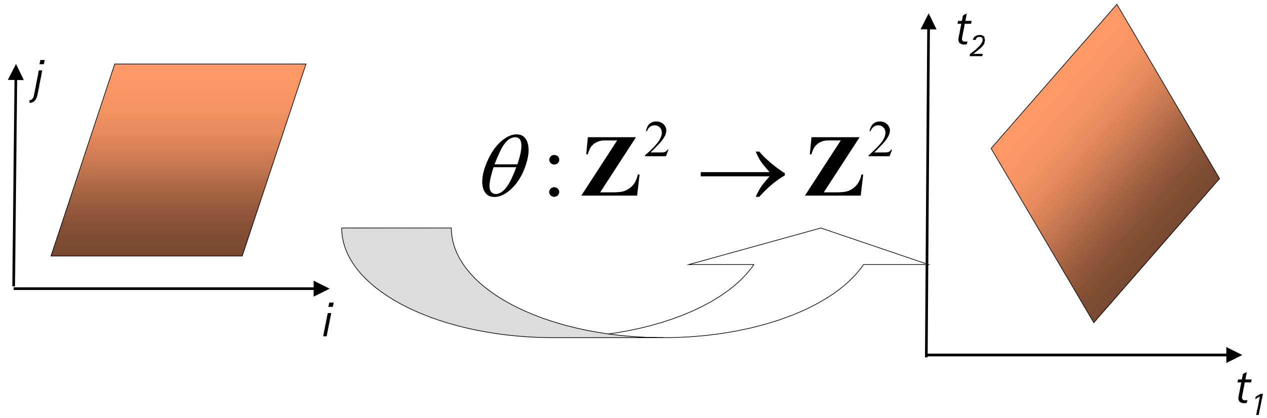
```
for(i=0; i<N; i++)
  for(j=0; j<N; j++)
    S1(i,j);
    S2(i,j)
```

$$\theta_1(i,j) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ j \\ 1 \end{bmatrix} \xrightarrow{\text{fusion}} \theta_1(i,j) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ j \\ 1 \end{bmatrix}$$

$$\theta_2(i,j) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \boxed{1} \\ 0 & 1 & 0 \\ 0 & 0 & \boxed{0} \end{bmatrix} \begin{bmatrix} i \\ j \\ 1 \end{bmatrix} \xleftarrow[\text{distribution}]{} \theta_2(i,j) = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \boxed{0} \\ 0 & 1 & 0 \\ 0 & 0 & \boxed{1} \end{bmatrix} \begin{bmatrix} i \\ j \\ 1 \end{bmatrix}$$

Loop transformations as scheduling

iteration space of a statement *S(i,j)*



Schedule θ maps iterations to multi-dimensional time

A feasible schedule must preserve dependencies

Loop transformations/synthesis mean generating code to execution iterations of a loop in the lexicographical order of time

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R-Stream: base affine scheduling and fusion

Generalization of Bondhugula's breakthrough algorithm in a new unified formulation

Model based on an *objective function* with several *cost coefficients*:

- slowdown in execution if a loop p is executed sequentially rather than in parallel
- cost in performance if two loops p and q remain unfused rather than fused

$$\textit{minimize} \left(\sum_{l \in \mathrm{loops}} w_l p_l + \sum_{e \in \mathrm{loopedges}} u_e f_e \right)$$

slowdown in sequential execution

cost of unfusing two loops

These two cost coefficients address parallelism and locality in a *unified and unbiased manner* (as opposed to traditional compilers)

Fine-grained parallelism, such as SIMD, can also be modeled using similar formulation

Balancing parallelism quality and memory usage

Enabling technologies:

- Exact dependence analysis and conservative approximations
- Violated dependence analysis
- Ability to reason about temporarily incorrect programs
- Automatic correction of loop transformations
- Polyhedral schedulers

Key ideas:

- Memory budget, autotunable
- Schedule aggressively (and wrongly)
- Correct by expansion (and index-set splitting)
- Need to support tiling (most important program transformation ever)

Algorithm – High-level ideas

Iterative fixed point algorithm: the problem is non-linear

Precisely pinpoint the sources of error (VDA supports tiling)

Expand to correct

If memory budget is exceeded, save the reason why

While there exist errors:

- Schedule using blackbox scheduler
- Plug-in saved dependences to constrain the scheduler
- Fixed-point is reached

Details in the paper

Optimization with BLAS vs. global optimization

Numerous cache misses

```
/* Optimization with BLAS */
for loop {
  ...
  BLAS call 1
  ...
  BLAS call 2
  ...
  ...
  BLAS call n
  ...
}
```

Outer loop(s)

Retrieve data Z from disk

Store data Z back to disk

Retrieve data Z from disk !!!

vs.

```
/* Global Optimization*/
doall loop {
  ...
  for loop {
    ...
    [read from Z]
    ...
    [write to Z]
    ...
    [read from Z]
  }
  ...
}
```

Can parallelize outer loop(s)

Loop fusion improves locality

→ Global optimization can expose better parallelism and locality

Parallelism/locality/memory tradeoff example

Array z gets expanded, to introduce another level of parallelism

Maximum distribution destroys locality

```
/*
 * Original code:
 *   Simplified CSLC-LMS
 */
for (k=0; k<400; k++) {
 for (i=0; i<3997; i++) {
  z[i]=0;
  for (j=0; j<4000; j++)
   z[i]= z[i]+B[i][j]*x[k][j];
 }
 for (i=0; i<3997; i++)
  w[i]=w[i]+z[i];
}
```

Max. parallelism (no fusion)

```
doall (i=0; i<400; i++)
 doall (j=0; j<3997; j++)
  z_e[j][i]=0
doall (i=0; i<400; i++)
 doall (j=0; j<3997; j++)
  for (k=0; k<4000; k++)
   z_e[j][i]=z_e[j][i]+B[j][k]*x[i][k];
doall (i=0; i<3997; i++)
 for (j=0; j<400; j++)
  w[i]=w[i]+z_e[i][j];
doall (i=0; i<3997; i++)
 z[i] = z_e[i][399];
```

Data accumulation

→ 2 levels of parallelism, but poor data reuse (on array z_e)

Parallelism/locality/memory tradeoff example (cont.)

```
/*
 * Original code:
 *   Simplified CSLC-LMS
 */
for (k=0; k<400; k++) {
  for (i=0; i<3997; i++) {
    z[i]=0;
    for (j=0; j<4000; j++)
      z[i]= z[i]+B[i][j]*x[k][j];
  }
  for (i=0; i<3997; i++)
    w[i]=w[i]+z[i];
}
```

Max. fusion →

Aggressive loop fusion destroys parallelism (i.e., only 1 degree of parallelism)

```
doall (i=0; i<3997; i++)
  for (j=0; j<400; j++) {
    z[i]=0;
    for (k=0; k<4000; k++)
      z[i]=z[i]+B[i][k]*x[j][k];
    w[i]=w[i]+z[i];
  }
```

→ Very good data reuse (on array z), but only 1 level of parallelism

Parallelism/locality/memory tradeoff example (cont.)

```
/*
 * Original code:
 *   Simplified CSLC-LMS
 */
for (k=0; k<400; k++) {
  for (i=0; i<3997; i++) {
    z[i]=0;
    for (j=0; j<4000; j++)
      z[i]= z[i]+B[i][j]*x[k][j];
  }
  for (i=0; i<3997; i++)
    w[i]=w[i]+z[i];
}
```

Parallelism with partial fusion

Expansion of array z

Partial fusion doesn't decrease parallelism

```
doall (i=0; i<3997; i++) {
  doall (j=0; j<400; j++) {
    z_e[i][j]=0;
    for (k=0; k<4000; k++)
      z_e[i][j]=z_e[i][j]+B[i][k]*x[j][k];
  }
  for (j=0; j<400; j++)
    w[i]=w[i]+z_e[i][j];
}
doall (i=0; i<3997; i++)
  z[i]=z_e[i][399];
```

Data accumulation

→ 2 levels of parallelism with good data reuse (on array z_e)

Interesting facts

Example is a very simplified 2-D from original 4-D problem

Parallelism / locality tradeoff is obtained by changing the cost model

- Coefficients that can be learnt, across programs, across architectures
- Multi-objective linear functions

Base algorithm is enough for good performance:

- Memory budget = infinity
- Minimal amount of expansion for the specified parallelism
- Much smaller than full static expansion (which does not fit in 8GB space)

Other programs are not that friendly:

- Degrade parallelism found by scheduler (set doall bit to 0)
- This produces fewer violations and less expansion

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R-Stream: Joint affine scheduling and fusion

R-Stream uses a heuristic based on an *objective function* with several *cost coefficients*:

- slowdown in execution if a loop *p* is executed sequentially rather than in parallel
- cost in performance if two loops *p* and *q* remain unfused rather than fused

$$\textit{minimize}\left(\sum_{l \in \text{loops}} \underbrace{w_l}_{\text{slowdown in sequential execution}} p_l + \sum_{e \in \text{loopedges}} \underbrace{u_e}_{\text{cost of unfusing two loops}} f_e\right)$$

These two cost coefficients address parallelism and locality in a *unified and unbiased manner* (as opposed to traditional compilers)

Fine-grained parallelism, such as SIMD, can also be modeled using similar formulation

Parallelism + locality + spatial locality + data layout

Hypothesis that auto-tuning should adjust these parameters

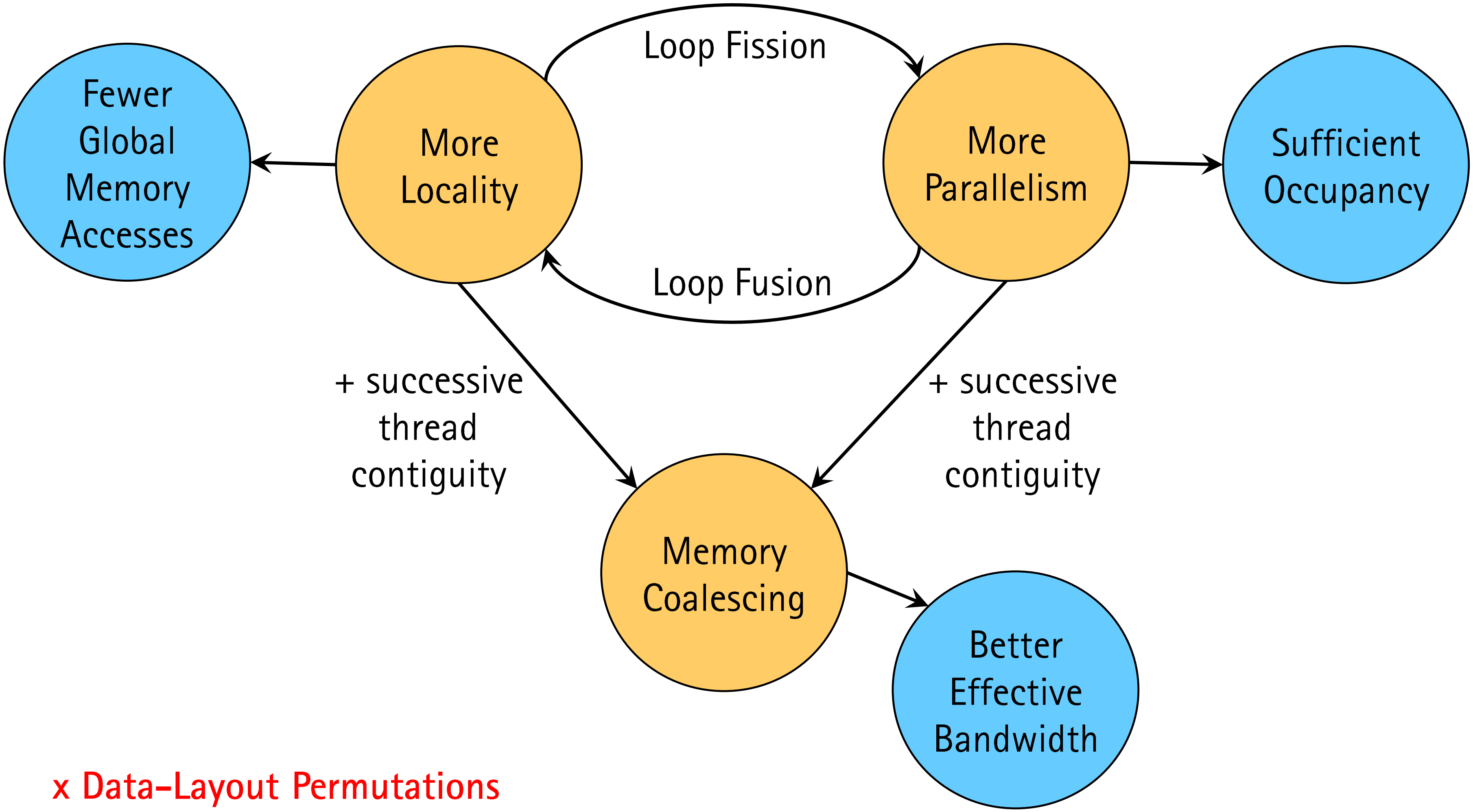
$$\sum_{l \in \text{loops}} w_l p_l + \sum_{e \in \text{loopedges}} u_e f_e$$

benefits of parallel execution

benefits of improved locality

New algorithm balances contiguity to enhance coalescing for GPU and SIMDization

Model for scheduling trades 3 objectives jointly



x Data-Layout Permutations
Additional degree of freedom

Joint affine scheduling and data layout

Enabling technologies:

- Generalization of Bondhugula's algorithm (Leung)
- Contiguity of a reference (Bastoul)
  - Generalization to any schedule dimension
  - Generalization to any array dimension
- Convex space of all legal multi-dimensional transformations
  - Need to bound the “alpha” variables
- Ability to write Im f <= Im A in a linear formulation
  - Linear when Im f = Im A (Leung)
  - Not exact linear when Im f < Im A

Algorithm – High-level ideas

Start from a multi-dimensional formulation

Incrementally add variables and constraints for more and more general formulations

- Contiguity for innermost schedule and array dimension
- Contiguity for any schedule and array dimension
- Contiguity constraints across all references in a statement
- Contiguity constraints for all statements "that have the same beta prefix"
- Mix with parallelism → simd and vectorization
  - no guarantee on strides in this paper
- Data layout permutations open new doors

Need an invertible solution:

- No magic bullet, depth by depth, heuristic strategies (not permutations)
- On the whole multi-dimensional problem

Joint affine scheduling and data layout

$$\forall\ \Delta = \{T \rightarrow S\},\ \forall\ k \in [1, min(d^S, d^T)],\ \forall\ (i^T, i^S) \in \Delta :$$

$$\begin{cases} \delta_k^\Delta \in \{0,1\} \\ \sum_{l=1}^{min(d^S,d^T)} \delta_l^\Delta = 1 \\ \Theta_k^T(i^T) - \Theta_k^S(i^S) \geq \\ \qquad -\mathcal{N}_\infty \left(\sum_{l=1}^{l \leq k-1} \delta_l^\Delta\right).(\vec{n}+1) + \delta_k^\Delta \end{cases}$$

Figure 1: Convex space of all legal schedules.

+ Im f <= Im a

$$c_{r,d}^F \in \{0,1\}$$

$$\mu - F_{\overline{r}} \cdot \lambda + \mathcal{N}_\infty \cdot (1 - c_{r,d}^F) \geq 0$$

$$-\mu + F_{\overline{r}} \cdot \lambda + \mathcal{N}_\infty \cdot (1 - c_{r,d}^F) \geq 0$$

+ Simd + data layout

$\forall S \in \mathcal{G},\ \forall l \in [1, d^S]$ $$\mathcal{K}_1^S \cdot \Sigma_l^S \leq \sum_{A \in S} p_l^{S,A}$$

$\forall S \in \mathcal{G},\ \forall l \in [1, d^S],\ \forall A \in S$ $$p_l^{S,A} \leq \sum_{r=1}^{dim\ A} q_{l,r}^{S,A}$$

$\forall S \in \mathcal{G},\ \forall l \in [1, d^S],\ \forall A \in S,$
$F\ accesses\ A$ $$\mathcal{K}_3^{S,A} \cdot q_{l,r}^{S,A} \leq \sum_{F\ acc.\ A} c_{l,r}^F$$

Inner contiguity, innermost array

```
for (i=1; i<=N; i++) {
  for (j=1; j<=N; j++) {
    for (k=1; k<=N; k++) {
      A[i-k][k]=A[i-k][k]+1;
}}}
```

```
for (i=1; i<=N; i++) {
  for (j=-N+1; j<=N-1; j++) {
    for (k=max(-j+1,1);
          k<=min(-j+N, N); k++) {
        A[-j][j+k]=A[-j][j+k]+1;
 }}}
```

→ (j, -i+k, i)
no contiguous solution in the positive quadrant

Outer vectorization innermost array dimension

```
for (i=2; i<=1+N; i++) {
  for (j=2; j<=1+M; j++) {
    for (k=1; k<=L; k++) {
      A[i][j][k]=A[i][j-1][k+1]+
        A[i-1][j][k+1];

}}}
```

```
doall (i=5; i<=N+M+L+2; i++) {
 for (j=max(2, i-N-L-1);
      j<=min(M+1, i-3); j++) {
   for (k=max(2, i-j-L);
     k<=min(i-j-1, N+1); k++) {
     A[k][j][i-j-k]=
       A[k][j-1][i-j-k+1]+
       A[k-1][j][i-j-k+1];
}}}
```

$\boxed{\rightarrow (i+j+k,\ j,\ k)}$

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Radar benchmarks (array expansion)

Beamforming algorithms:

- **MVDR-SER:** Minimum Variance Distortionless Response using Sequential Regression
- **CSLC-LMS:** Coherent Sidelobe Cancellation using Least Mean Square
- **CSLC-RLS:** Coherent Sidelobe Cancellation using Robust Least Square

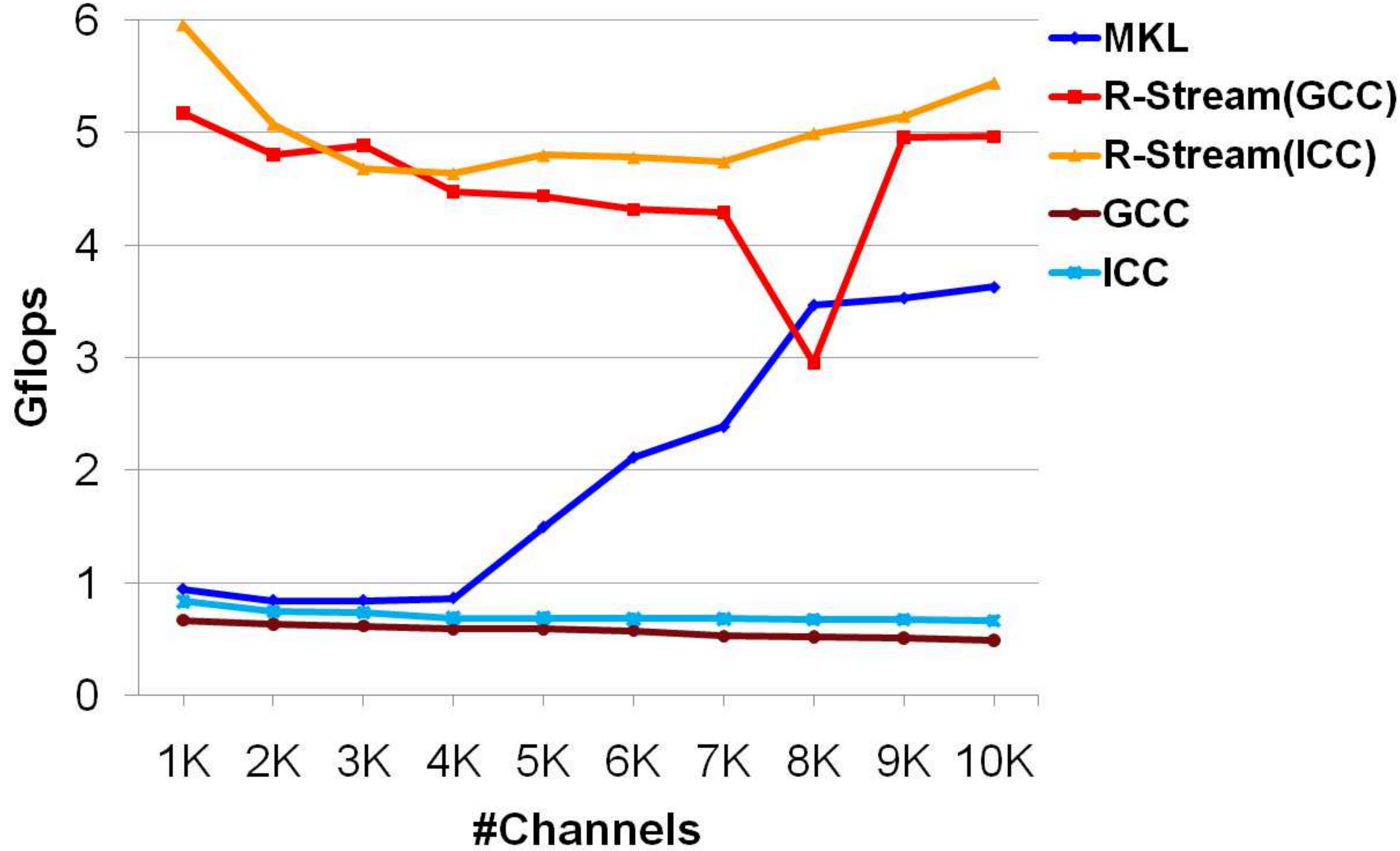
Expressed in sequential ANSI C

400 radar iterations

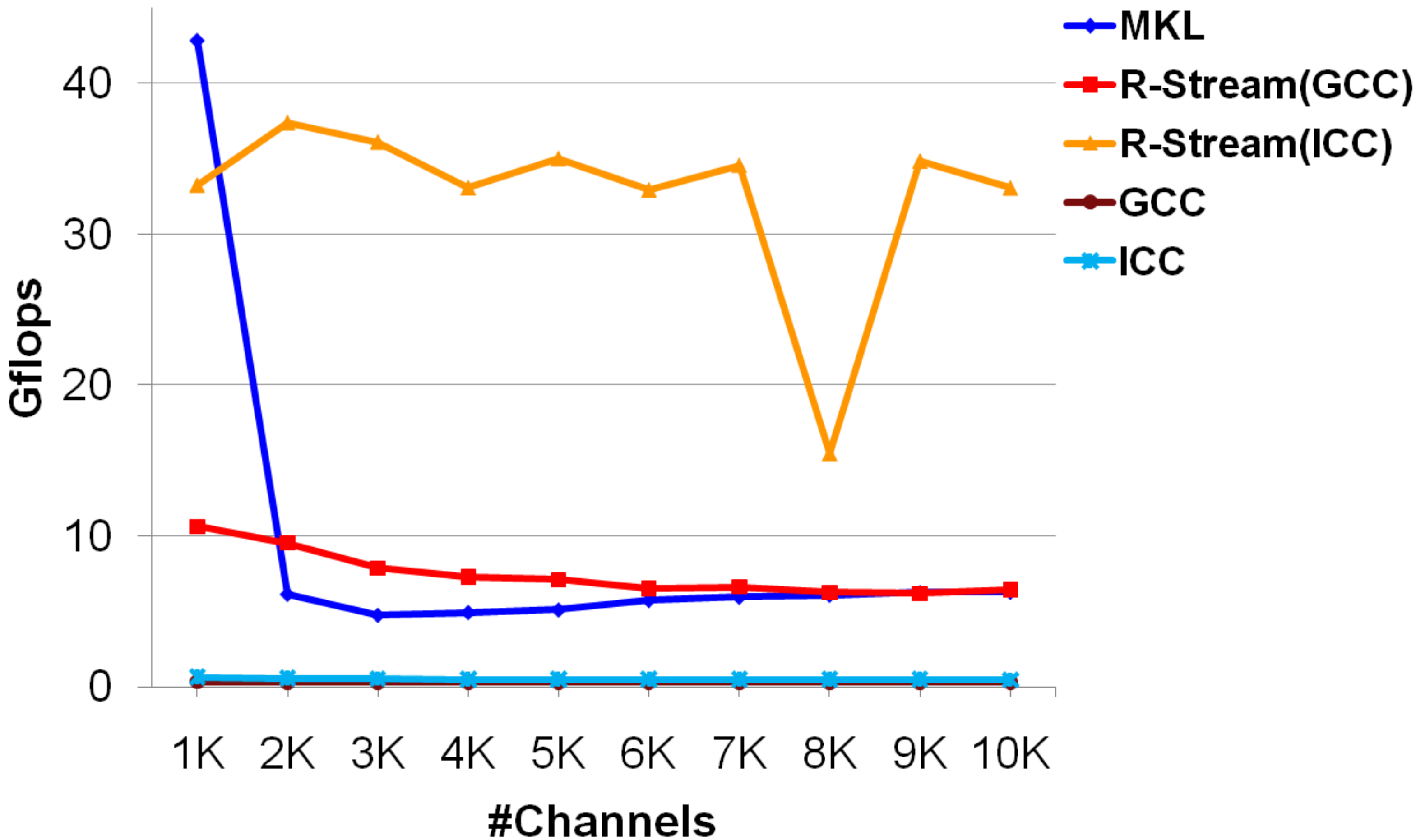
Compute 3 radar sidelobes (for CSLC-LMS and CSLC-RLS)

The problem is algorithm selection: which of these 3 algorithms has the most parallelism.

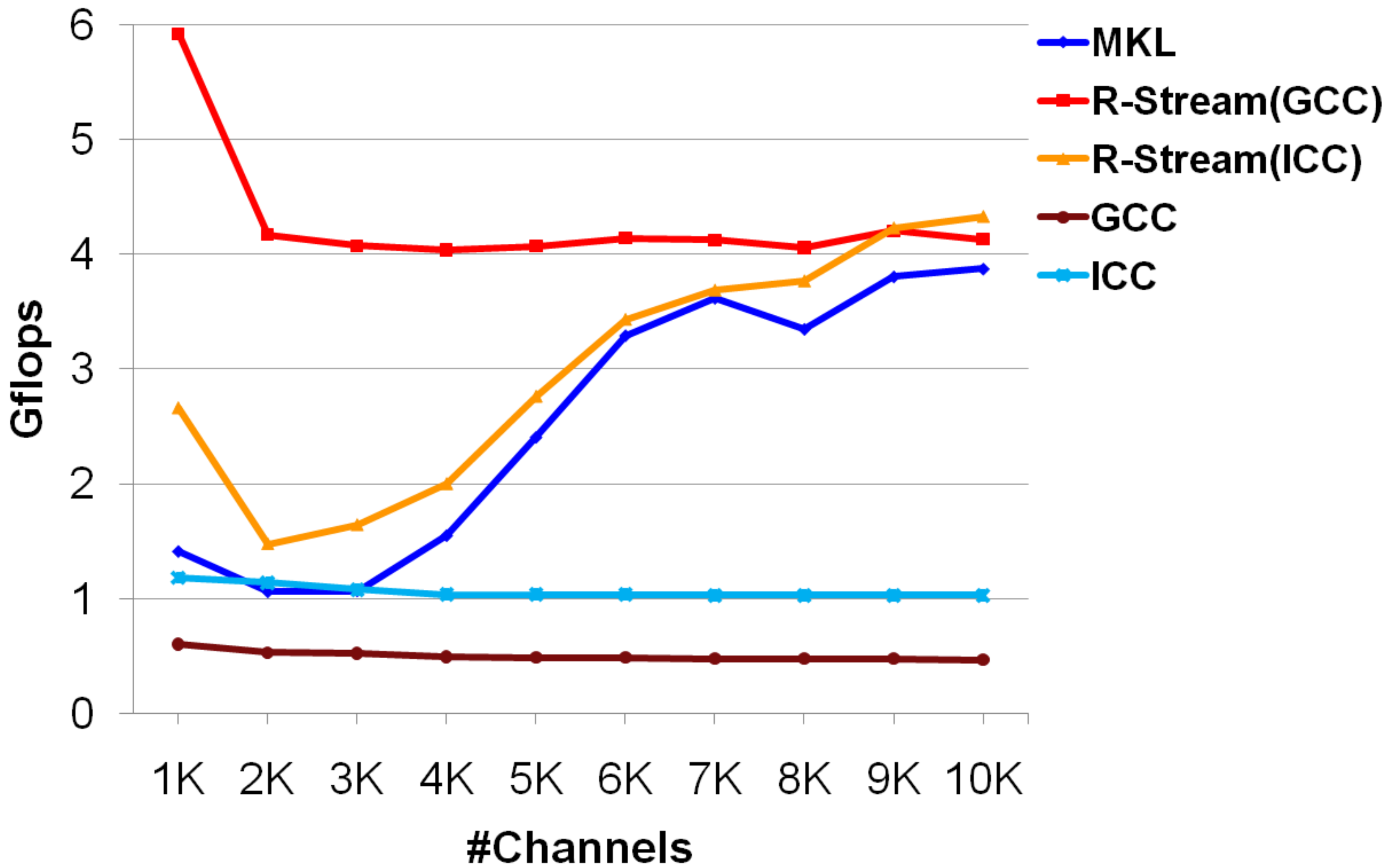
MVDR-SER (outer-sequential (array expansion))



CSLC-LMS (outer-parallel (array expansion))



CSLC-RLS (outer-sequential (array expansion))



Vectorization quality (statistical results)

Strategy	Num contiguous	Num simd	Simd depth	Num T/O
NoObj	-	-	-	2
Identity	179	23	27	2
Permutations	673	75	119	2
Default	1637	386	586	2
OuterSimd	2891	348	418	2
Layout	2107	483	772	2
OuterSimd + Layout	6999	368	244	3
AS	-	-	-	0

400 kernel benchmarks

Includes some of the PolyBench

Includes multiple larger “apps” from radar world

Scalability limitations many possible formulation improvements

Applied on whole problem when you would typically apply within a tile

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New formulations that now need to be tuned and scaled up

- UTVPI direction is interesting

Further opportunities to integrate even more transformations into iterative, fixed-point algorithms

- Contraction, ISS
- But the problem is non-trivial because of placement, synchronizations and communications
- Global problem not yet understood well enough

Algorithm selection exploration

Autotuning at every level in the compiler: built but not yet exploited

Modelization of energy constraints

- Will likely require folding in placement + privatization in scheduling somehow

Conclusion

Still lots of opportunities

- At low-level we compare auto-tuned MKL (human + tools) to fully auto-generated high-level C
- Soon able to model energy consumption

The next frontier is integration with data structures and ADTs

Research collaborations

- More tools to explore and ideas than people at Reservoir

Questions?