On the Variety of Static Control Parts in Real-World Applications: from Affine via Multi-dimensional to Polynomial and Just-in-Time

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4th International Workshop on Polyhedral Compilation Techniques

Defining the Real World

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- LLVM (`llvm.org`)

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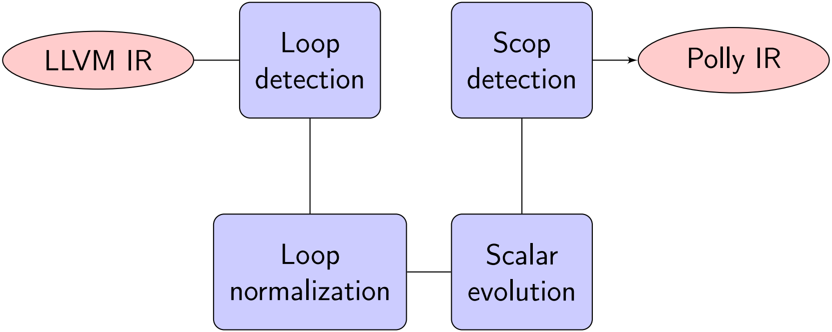
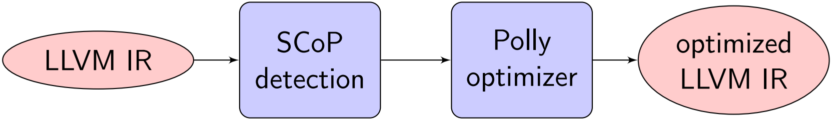
- Polly (polly.llvm.org)
- PolyJIT (www.infosun.fim.uni-passau.de/cl/PolyJIT)

Automatic Detection of SCoPs in LLVM



Effectiveness of Automatic Polyhedral Optimization



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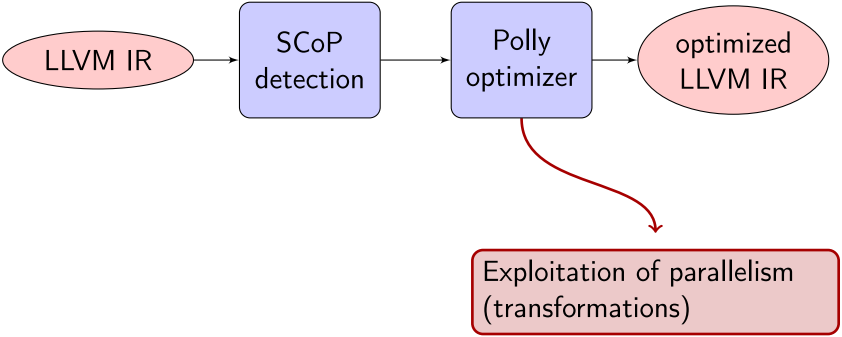

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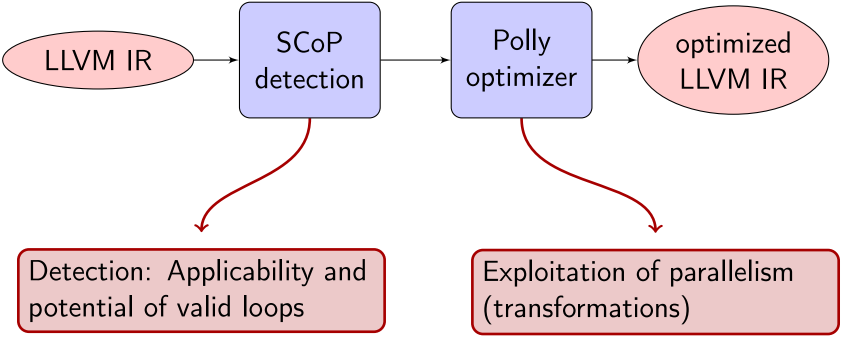

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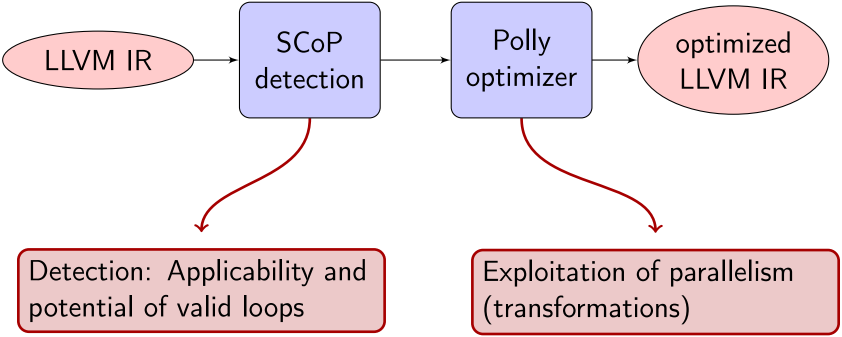

The detection process lacks thorough empirical evaluation!

PolyJIT: pprof

- Set of 50 programs commonly used in various domains.
- 8 domains (Multimedia, Scientific, Simulation, Encryption, Compilation, Compression, Databases, Verification).
- Extract run time and compile time statistics.

Measuring a SCoP's fraction of the total run time

What fraction of a program's total run time is spent inside SCoPs?

```
for (int i=0; i<=n; ++i)
  for (int j=i; j<=n; ++j)
    if (i >= n-j) {
S: A[i+n][j+i] = B[n+2*i-1][j];
T: B[i+n][j-i] = A[n-2*i+1][j];
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```

Definition (Execution SCoP coverage)

$$\text{ExecCov} = \frac{\text{Time spent inside SCoPs}}{\text{Total program run time}}$$

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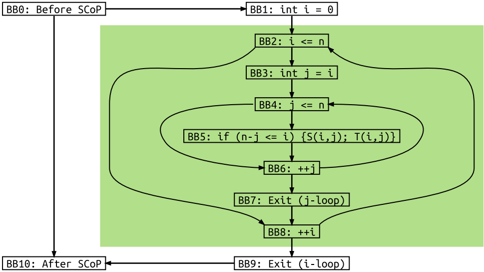

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Static Control Parts: Class Static

Detection at compile time

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1. Affine expressions in
   - Loop bounds
   - Conditions
   - Memory accesses

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1. Affine expressions in
   - Loop bounds
   - Conditions
   - Memory accesses
2. Static control flow
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What can we do, if it is not a static (affine) SCoP?

Problem 1: Multi-dimensional array accesses

Contiguous

```
A[i][j];
```

Problem 1: Multi-dimensional array accesses

Contiguous

clang -O0

```
%0 = mul nsw i32 %i, %n
%idx = getelementptr float* %A, i32 %0
%idx1 = getelementptr float* %idx, i32 %j
```

`A[i][j]`

Problem 1: Multi-dimensional array accesses

Contiguous

clang -O1

```
%0 = mul nsw i32 %i, %n
%idx.s = add i32 %0, %j
%idx1 = getelementptr float* %A, i32 %idx.s
```

`A[n*i+j]`

Problem 1: Multi-dimensional array accesses

Contiguous

clang -O1

```
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Delinearization of array accesses

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A[n*i+i+j]
```

$$n * i + i + j = (n + 1) * i + j$$

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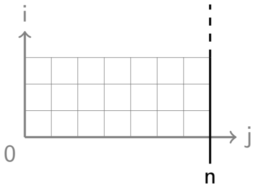

Delinearization of array accesses

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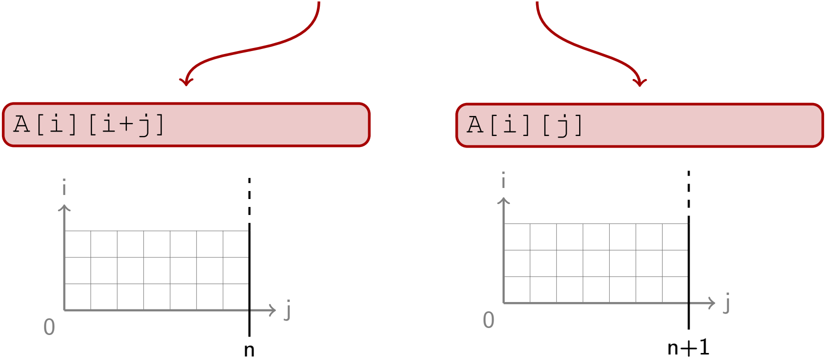

Static Control Parts: Class Multi

Let's allow delinearizeable accesses!

```
A[(n+2+m)*i]                A[i'+2*m+2*n]
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Static Control Parts: Class Multi

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`A[(n+2+m)*i]`	`A[i'+2*m+2*n]`
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$a = a'$ $\qquad ni + 2i + mi = i' + 2m + 2n$

$a - a' = 0$ $\qquad ni + 2i + mi - i' - 2m - 2n = 0$

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Split into terms	$ni, 2i, mi, -i', -2m, -2n$

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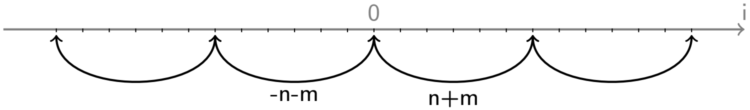

Bounds check	$\lvert 1(2i - i') \rvert \leq \lvert n + m \rvert - 1$

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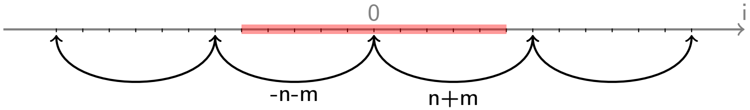

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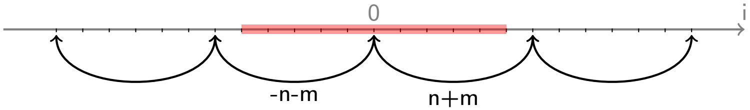

Bounds check $|1(2i - i')| \leq |n+m| - 1$

$$a - a' = 0 \Leftrightarrow i - 2 = 0 \wedge 2i - i' = 0$$

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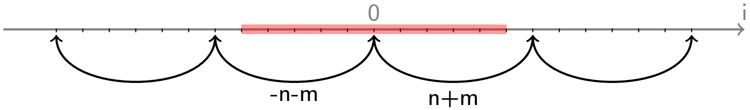

Bounds check $\quad |1(2i - i')| \leq |n+m| - 1$

$$a - a' = 0 \Leftrightarrow i - 2 = 0 \wedge 2i - i' = 0$$

$$i = 2 \text{ and } i' = 4$$

Static Control Parts: Class Multi

Let's allow delinearizeable accesses!

$$a - a' = \sum_{x=1}^{k} \pi_x \gamma_x \tag{1}$$

Where π_x are polynomials in the parameters and γ_x are affine expressions in the iterators.

$$\forall \vec{i} \in D : |\pi_x \gamma_x| \leq |\pi_{x+1}| - 1 \text{ for } 1 \leq x < k \tag{2}$$

When (1) and (2) hold, $a - a' = 0$ is equivalent to

$$\gamma_1 = 0 \wedge \cdots \wedge \gamma_k = 0$$

Static Control Parts: Class Algebraic

Let's allow polynomials!

```
for (int i=0; i<=n; i++) {
  A[m*i+n] = __;
  __ = A[m*(i-1)+n];
}
```

Static Control Parts: Class Algebraic

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for (int i=0; i<=n; i++) {
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Accept arbitrary polynomials in

- Loop bounds
- Array subscripts
- Unsupported: Products in the iterators (`i*i`)

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Accept arbitrary polynomials in

- Loop bounds
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Multi is a subset of *Algebraic*

Problem 2: Multi-dimensional array accesses

Non-Contiguous

```
float **A;
A[i][j] = A[i-1][j-1];
```

Problem 2: Multi-dimensional array accesses

Non-Contiguous

```
%i = load i64* %i.addr
%j = load i64* %j.addr
%outer = load float*** %A
%arrayidx3 = getelementptr inbounds float**
            %outer, i64 %i
%inner = load float** %arrayidx3
%arrayidx4 = getelementptr inbounds float*
            %inner, i64 %j
```

Static Control Parts: Class Pointer to Pointer

Let's allow pointers to pointers!

```
float **A;
for (int i=0; i<=n; ++i)
  for (int j=i; j<=n; ++j)
    A[i][j] = A[i-1][j-1];
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- No aliasing between inner dimensions.

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  for (int j=i; j<=n; ++j)
    A[i][j] = A[i-1][j-1];
```

- No aliasing between inner dimensions.
- No aliasing of the outer dimension with other pointers/arrays.

Static Control Parts: Class Dynamic

Let's be lazy and do everything at run time

```
for (int i=0; i<=n; i++) {
  A[m*i+n] = __;
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}
```

Static Control Parts: Class Dynamic

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```
for (int i=0; i<=n; i++) {
  A[m*i+n] = __;
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}
```

- Run time specialization for

Static Control Parts: Class Dynamic

Let's be lazy and do everything at run time

```
for (int i=0; i<=n; i++) {
  A[42*i+n] = __;
  __ = A[42*(i-1)+n];
}
```

- Run time specialization for
  - Known parameter values

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- Run time specialization for
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for (int i=0; i<=n; i++) {
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- Run time specialization for
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- Run time specialization for
  - Known parameter values
  - Known aliasing
- Function calls to other SCoPs

Expectations

Compile time

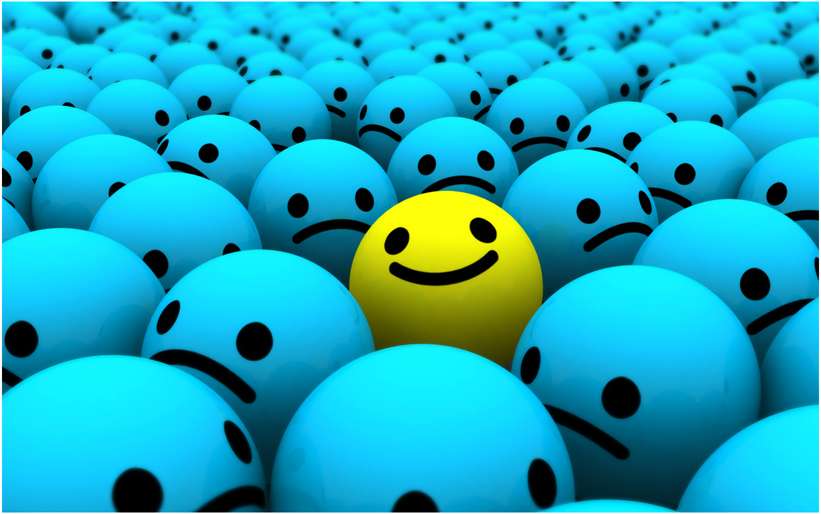
- Multi-dimensional array accesses are used often, so *Multi* (*Algebraic*) should contain a lot more SCoPs than *Static*.
- *Pointer to Pointer* should cover a few more SCoPs than *Static*.

Run time

$$\textit{Static} \subseteq \textit{Multi} \subseteq \textit{Algebraic} \subseteq \textit{Dynamic}$$

$$\textit{Static} \subseteq \textit{Pointer to Pointer} \subseteq \textit{Pointer to Pointer (No Alias)}$$

Reality



Reality

Name	Stat | Alg | Dyn	S_{st}
Compilation		
js		120
python		84
ruby		100
tcc		18
Compression		
7za		100
bzip2		30
gzip		23
xz		23
Multimedia		
avconv		950
povray		110
x264		55
Scientific		
linpack		9
xeigtstc		630
xeigtstd		610
xeigtsts		610
xlintstd		690
xlintstds		160
xlintstrfc		170
xlintstrfd		150
xlintstrfs		150
xlintstrfz		170
xlintsts		530
xlintstzc		190

Name	Stat | Alg | Dyn	S_{st}
Encryption		
blowfish		0
bn		24
cast		2
ccrypt		3
des		9
dsa		24
ecdsa		24
hmac		24
mcrypt-aes		38
mcrypt-cip		39
md5		24
openssl		67
rc4		1
rsa		24
sha1		24
sha256		24
sha512		24
ssl		32
Simulation		
crafty		57
lammps		330
lulesh-omp		16
lulesh		14

Findings

Class *Algebraic* and *Multi*

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- Very low increment in SCoPs (between 2 and 37) over *Static*.

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- Povray shows an $ExecCov_{Algebraic}$ of 41% as compared to an $ExecCov_{Static}$ of 8.5%.

Findings

Class *Algebraic* and *Multi*

- Very low increment in SCoPs (between 2 and 37) over *Static*.
- Only 10 out of 50 experiments show a small number of non-affine expressions
- Povray shows an $ExecCov_{Algebraic}$ of 41% as compared to an $ExecCov_{Static}$ of 8.5%.
- No notable increase in $ExecCov_{Algebraic}$ in the other experiments.

Findings

Name	S_{st}	S_{pp}	S_{ppa}
Compilation			
js	120	36	95
python	84	6	33
ruby	100	19	40
tcc	18	0	0
Compression			
7za	100	9	24
bzip2	30	0	0
gzip	23	0	0
xz	23	1	1
Multimedia			
avconv	950	11	310
povray	110	3	71
x264	55	14	44
Scientific			
linpack	9	0	0
xeigtstc	630	5	5
xeigtstd	610	0	0
xeigtsts	610	0	0
xlintstd	690	0	0
xlintstds	160	0	0
xlintstrfc	170	0	0
xlintstrfd	150	0	0
xlintstrfs	150	0	0
xlintstrfz	170	0	0
xlintsts	530	0	0
xlintstzc	190	0	0

Name	S_{st}	S_{pp}	S_{ppa}
Encryption			
blowfish	0	0	0
bn	24	1	10
cast	2	0	0
ccrypt	3	0	0
des	9	0	7
dsa	24	1	10
ecdsa	24	1	10
hmac	24	1	10
mcrypt-aes	38	0	0
mcrypt-cip	39	0	0
md5	24	1	10
openssl	67	1	18
rc4	1	0	0
rsa	24	1	10
sha1	24	1	10
sha256	24	1	10
sha512	24	1	10
ssl	32	1	12
Simulation			
crafty	57	0	9
lammps	330	45	470
lulesh-omp	16	0	0
lulesh	14	0	0

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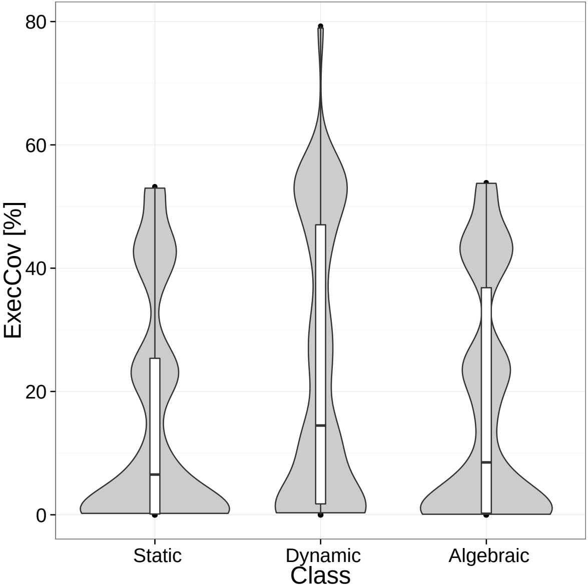
- The SCoP count can be (optimistically) increased by disabling alias checks.

Findings

Class *Pointer to Pointer*

- Low increment in SCoPs compared to *Static* (between 1 and 45). 22 out of 50 experiments show an increment in SCoP count.
- The SCoP count can be (optimistically) increased by disabling alias checks.
- No $\mathit{ExecCov}_{\mathit{Pointer\ to\ Pointer}}$ information available yet.

Run-time findings



Threats to Validity

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1. Construct Validity: Timing causes overhead.

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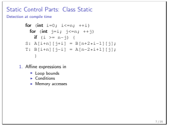
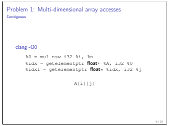
2. External Validity: Generalizability depends on the sample size.
3. Internal Validity: Quality of the input data. Relying on developer's testing.

Questions?

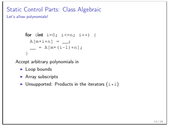
The Real World



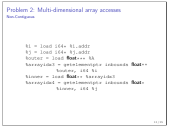
SCoPs: Static



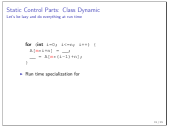
SCoPs: Multi



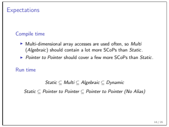
SCoPs: Algebraic



SCoPs: Pointer to Pointer



SCoPs: Dynamic



Expectations

Reality

Static Control Parts: Class Multi

Let's allow delinearizeable accesses!

1. Split $a - a'$ into its terms, i.e., $a - a' = \sum_{x=1}^{l} t_x$ where each t_x is a product of iterators and parameters (and a constant).
2. Group terms by their parameters, i.e., $a - a' = \sum_{x=1}^{m} \rho_x \gamma_x$ where each ρ_x is a product of parameters (or a constant).
3. Factor out common γ_x, i.e., $a - a' = \sum_{x=1}^{k} \pi_x \gamma_x$
4. Check the (total) ordering criterion using quantifier elimination, i.e.,

$$\forall \vec{i}, \vec{i'}, \vec{p} (\vec{i} \in D \wedge \vec{i'} \in D' \rightarrow |\pi_x \gamma_x| \leq |\pi_y| - 1)$$