Live-Range Reordering

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Outline

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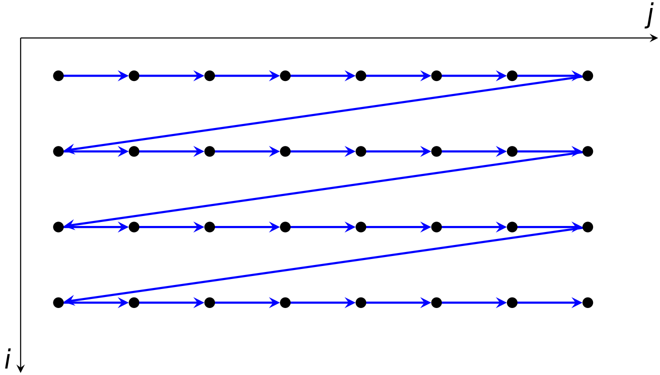
Tiling Intuition



Assume reuse along rows and columns

→: execution order

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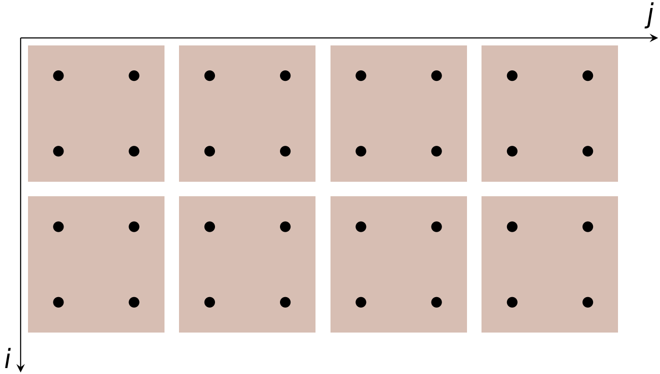

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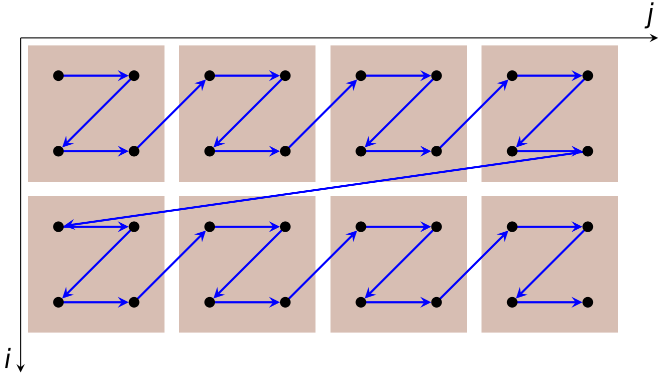

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Tiling Example

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for (i = 0; i < m; i++)
  for (j = 0; j < n; j++) {
    temp2 = 0;
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    }
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(symm.c from PolyBench/C 4.1)

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After tiling:

```
for (int c0 = 0; c0 < m; c0 += 32)
  for (int c1 = 0; c1 < n; c1 += 32)
    for (int c2 = 0; c2 <= min(31, m - c0 - 1); c2 += 1)
      for (int c3 = 0; c3 <= min(31, n - c1 - 1); c3 += 1) {
        temp2 = 0;
        for (int c4 = 0; c4 < c0 + c2; c4 += 1) {
          C[c4][c1 + c3] += ((alpha * B[c0 + c2][c1 + c3]) * A[c0 + c2][c4
          temp2 += (B[c4][c1 + c3] * A[c0 + c2][c4]);
        }
        C[c0 + c2][c1 + c3] = (((beta * C[c0 + c2][c1 + c3]) + ((alpha * E
      }
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Schedule Constraints

Tiling is a form of restructuring loop transformation

$\Rightarrow$ changes execution order of statement instances

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- ⇒ impose schedule constraints of the form

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In particular, any statement instance writing a value should be executed before any statement instance reading that value

- ⇒ *flow dependences* aka *live ranges*

Moreover, no write from **before** or **after** the live-range should be moved **inside** the live-range

- ⇒ traditionally,
  - *output dependences* between two writes to same location
  - *anti-dependences* between reads and subsequent writes to same location

Schedule Constraints Example

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avg = 0.f;
for (i=0; i<N; ++i)
  avg += A[i];
avg /= N;
for (i=0; i<N; ++i) {
  tmp = A[i] - avg;
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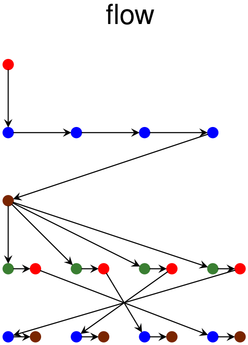

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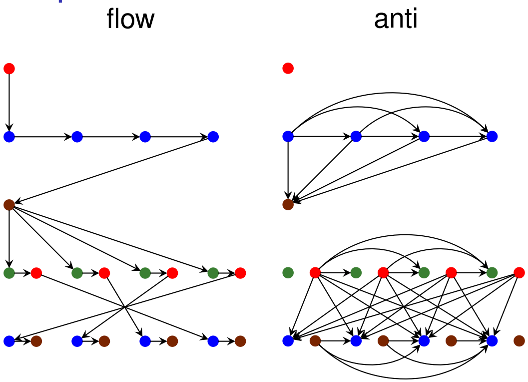

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- ⇒ anti-dependence between every instance of statement reading `temp2` and every later instance writing to `temp2`
- ⇒ serialized execution order

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$\Rightarrow$ anti-dependence between every instance of statement reading `temp2` and every later instance writing to `temp2`

$\Rightarrow$ serialized execution order

Such serializing anti-dependences are very common in practice

$\Rightarrow$ occur in nearly all experiments of Baghdadi, Beaugnon, et al. (2015)

$\Rightarrow$ no optimization possible without alternative to anti-dependences

Outline

1. Introduction
   - Example
   - Schedule Constraints

2. Live Range Reordering
   - Related Work
   - Scheduling
   - Relaxed Permutability Criterion
   - Conditional Validity Constraints

3. Conclusion

Alternatives to Anti-Dependences

- Conversion to single assignment through expansion (possibly followed by contraction)
  - \+ full scheduling freedom
  - (–) may increase memory requirements

Note: choice also has effect on scheduling time

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After expansion:

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for (i = 0; i < m; i++)
  for (j = 0; j < n; j++) {
    temp2[i][j][0] = 0;
    for (k = 0; k < i; k++) {
       C[k][j] += alpha*B[i][j] * A[i][k];
       temp2[i][j][k+1] = temp[i][j][k] + B[k][j] * A[i][k];
    }
    C[i][j] = beta*C[i][j] + alpha*B[i][j]*A[i][i] + alpha*temp2[i][j][i];
}
```

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- Conversion to single assignment through expansion (possibly followed by contraction)
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- Cluster live-range statements
  Note:
  - in general, clustering is partial scheduling
  - simple clusterings lead to coarse statements
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- Live-range reordering
  - + no increase in memory requirements
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Live-Range Reordering

Basic idea:

allow live-ranges to be reordered with respect to each other as long as they do not overlap

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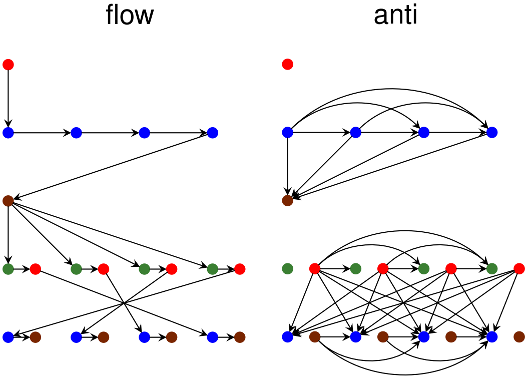

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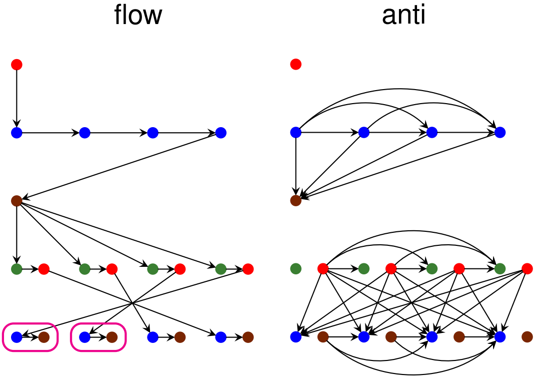

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- relaxed permutability criterion (Baghdadi, Cohen, et al. 2013)

  application by Baghdadi, Cohen, et al. (2013):
  - use standard scheduling algorithm
  - *reinterpret* results
- variable liberalization (Mehta 2014)
  - removes specific patterns of anti-dependences
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Scheduling

A schedule determines the *execution order* of statement instances and is expressed using a (recursive) combination of

- affine functions f
  $f(\mathbf{i}) < f(\mathbf{j}) \quad \Rightarrow \mathbf{i}$ executed before $\mathbf{j}$
- finite sequence $S_1, S_2, \ldots, S_n$
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Scheduling determines schedule compatible with schedule constraints

statement instance $\mathbf{a}$ needs to be executed before instance $\mathbf{b}$

$\Rightarrow$ there is some node with

$f(\mathbf{a}) < f(\mathbf{b}) \quad$ or $\quad \mathbf{a} \in S_{k_1} \wedge \mathbf{b} \in S_{k_2} \wedge k_1 < k_2$

$\Rightarrow$ for all outer nodes

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Band: nested sequence of affine functions that can be freely reordered

Scheduling Example 1

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for (i = 1; i < n; ++i)
A:M[i, 0] = f();
for (i = 1; i < n; ++i)
B:M[0, i] = g();
for (i = 1; i < n; ++i)
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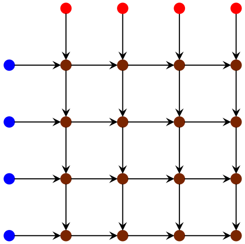

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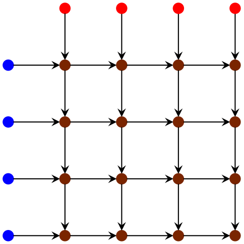

Schedule

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$|$

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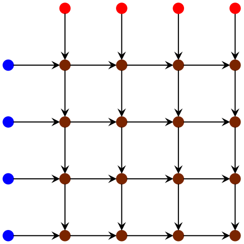

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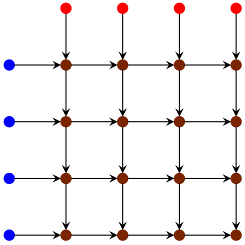

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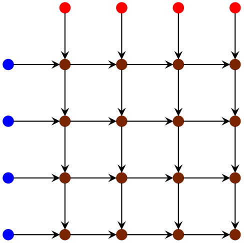

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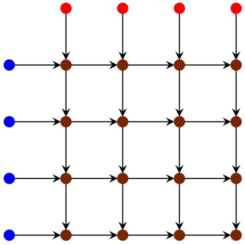

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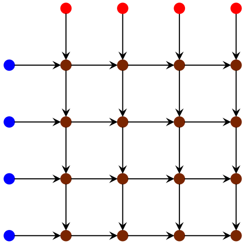
$\mathtt{B}[i] \to \mathtt{C}[0,i]$	$0 \to 0$	$i \to i$
$\mathtt{C}[i,j] \to \mathtt{C}[i+1,j]$	$i \to i+1$	$j \to j$
$\mathtt{C}[i,j] \to \mathtt{C}[i,j+1]$	$i \to i$	$j \to j+1$

Scheduling Example 1

```
for (i = 1; i < n; ++i)
A:M[i, 0] = f();
for (i = 1; i < n; ++i)
B:M[0, i] = g();
for (i = 1; i < n; ++i)
  for (j = 1; j < n; ++j)
C:  M[i][j] = h(M[i-1][j], M[i][j-1]);
```



Schedule

$\mathtt{A}[i] \to i; \mathtt{B}[i] \to 0; \mathtt{C}[i,j] \to i$

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$\{ \mathtt{A}[i] \}, \{ \mathtt{B}[i] \}, \{ \mathtt{C}[i,j] \}$

Schedule constraints

$\mathtt{A}[i] \to \mathtt{C}[i,0]$	$i \to i$	$0 \to 0$
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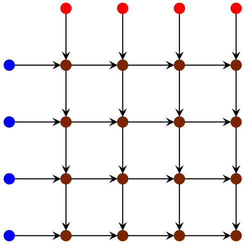

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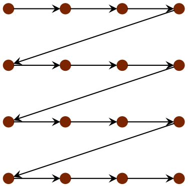
Scheduling Example 2

```
for (i = 0; i < n; ++i)
  for (j = 0; j < n; ++j)
S:   t = f(t, A[i][j]);
```



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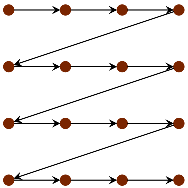

Schedule

$S[i,j] \rightarrow i$

$S[i,j] \rightarrow j$

Schedule constraints

$S[i,j] \rightarrow S[i,j+1]$

$S[i,n-1] \rightarrow S[i+1,0]$

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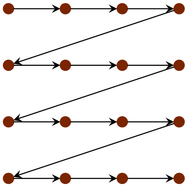

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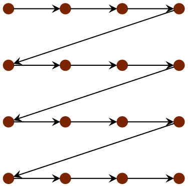

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Schedule constraints

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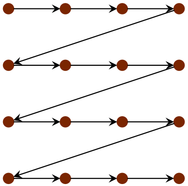

Schedule

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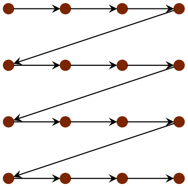

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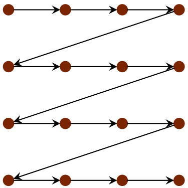

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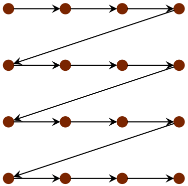

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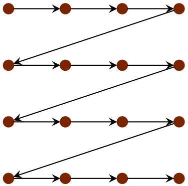

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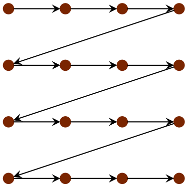

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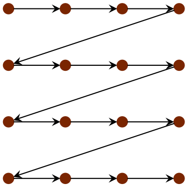

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Schedule constraints

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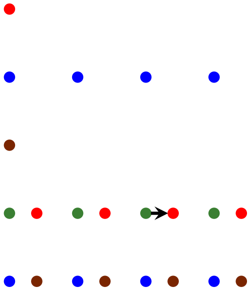

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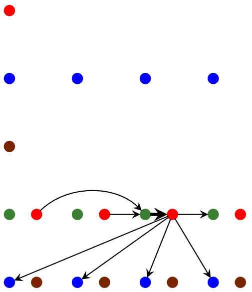

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  An anti-dependence is *adjacent* to a live-range if the source of one is the sink of the other
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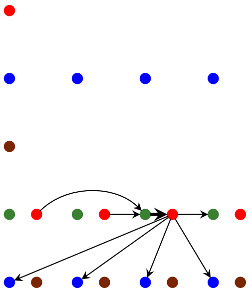
  A live-range is *local* to a band if its source and sink are assigned the same value by all affine functions in the band



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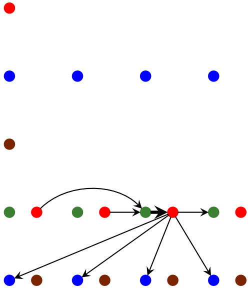
  If an anti-dependence is only adjacent to live-ranges that are local to a band,
  then the anti-dependence can be ignored within the band



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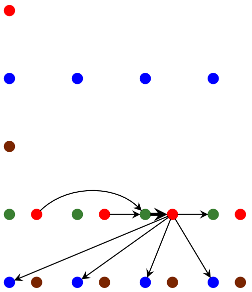

Baghdadi, Cohen, et al. (2013) use criterion to *reinterpret* schedule

⇒ combine nested sequences of bands after schedule construction

Conditional Validity Constraints

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  – condition $\rightarrow$ live-ranges
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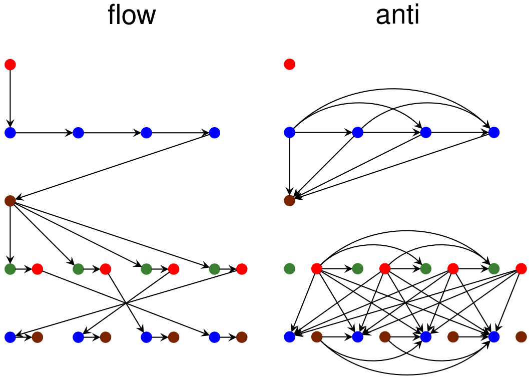
    or
  - – adjacent conditional validity constraints are satisfied → adjacent anti-dependences
- Conditional validity constraints handled during schedule construction
  - ignore conditioned validity constraints during band member computation
  - compute violated conditioned validity constraints
  - compute adjacent conditions
  - force adjacent conditions to be local in subsequent band members
  - recompute band if not local in current or previous members

Schedule Constraints Example

```
avg = 0.f;
for (i=0; i<N; ++i)
  avg += A[i];
avg /= N;
for (i=0; i<N; ++i) {
  tmp = A[i] - avg;
  A[i] = tmp;
}
for (i=0; i<N; ++i) {
  tmp = A[N - 1 - i];
  B[i] = tmp;
}
```



$\{ S0[]; S1[i]; S2[] \}, \{ S3[i]; S4[i]; S5[i]; S6[i] \}$

$S0[] \to 0; S1[i] \to i; S2[] \to N-1$

$S3[i] \to i; S5[i] \to N-1-i;$

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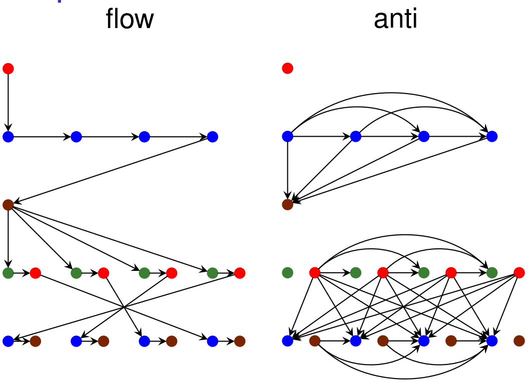

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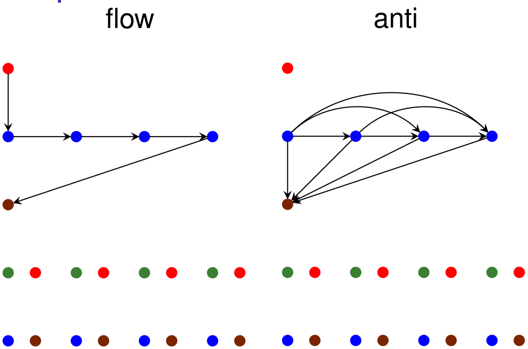

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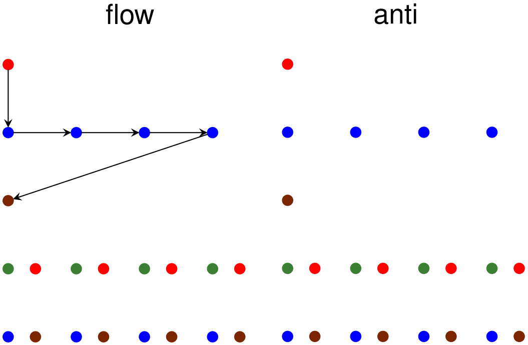

$\{\,\mathtt{S0}[\,];\mathtt{S1}[i];\mathtt{S2}[\,]\,\},\{\,\mathtt{S3}[i];\mathtt{S4}[i];\mathtt{S5}[i];\mathtt{S6}[i]\,\}$

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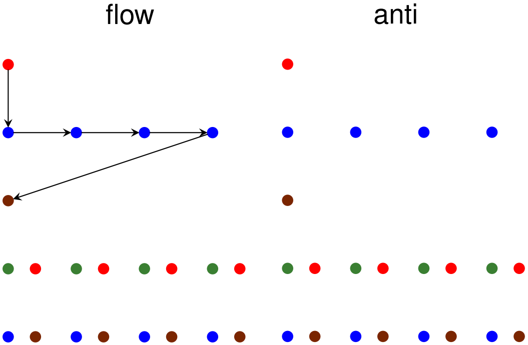

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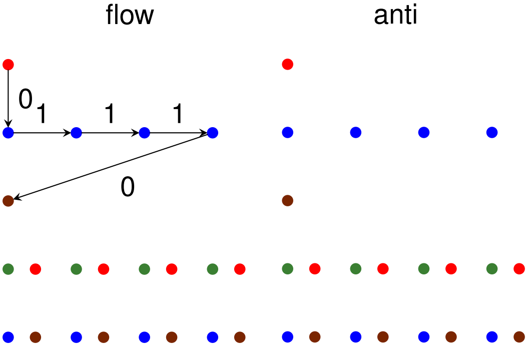

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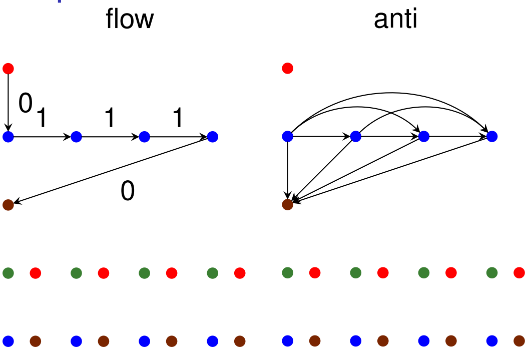

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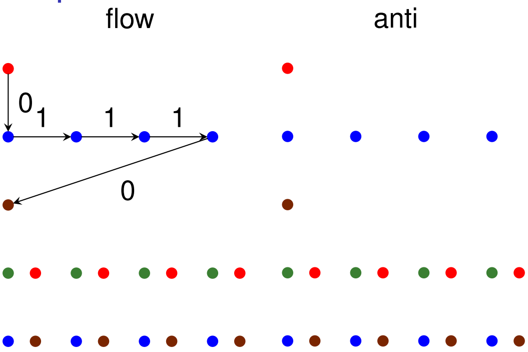

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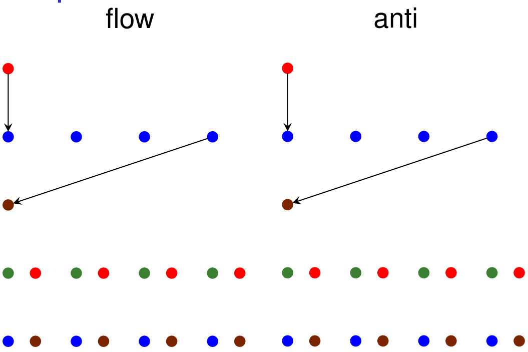

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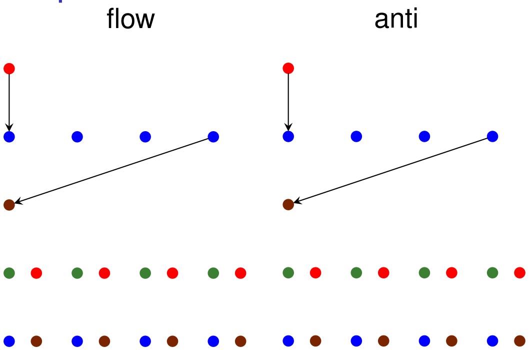

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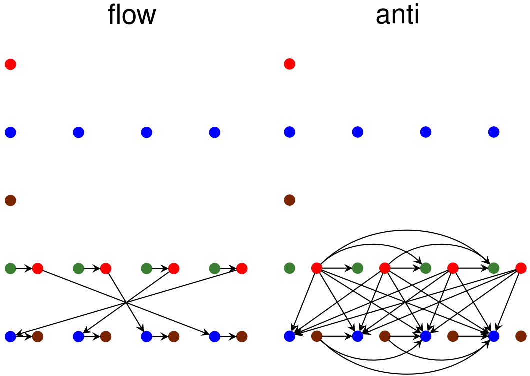

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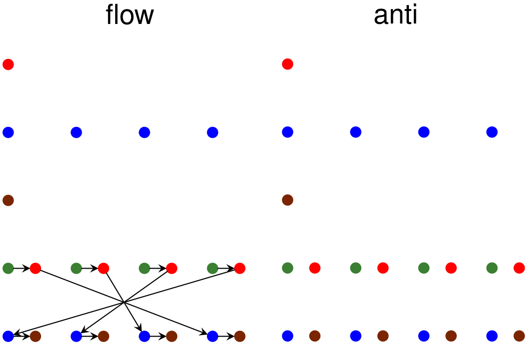

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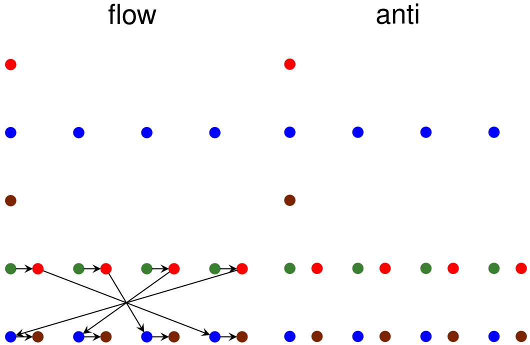

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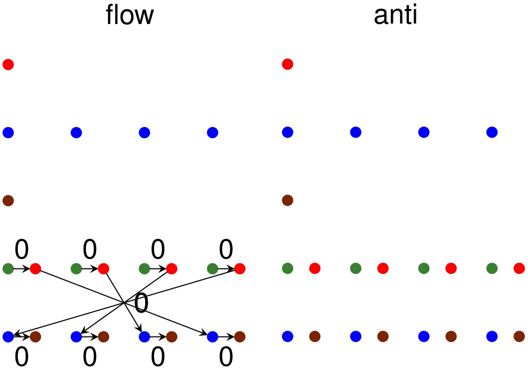

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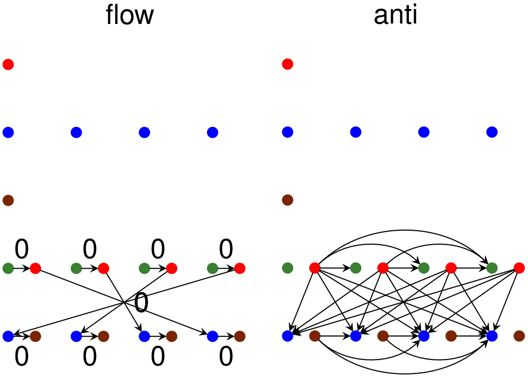

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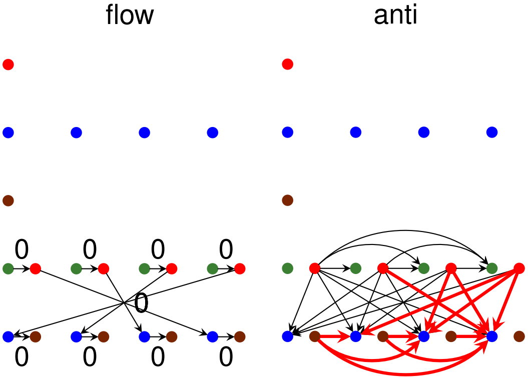

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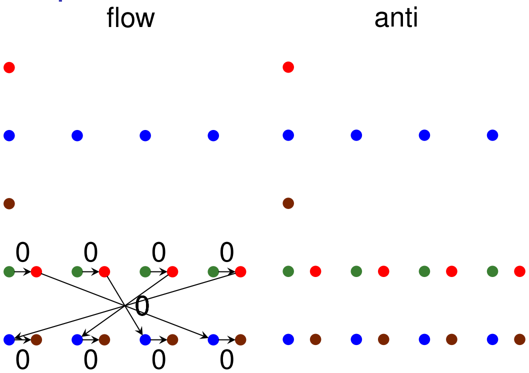

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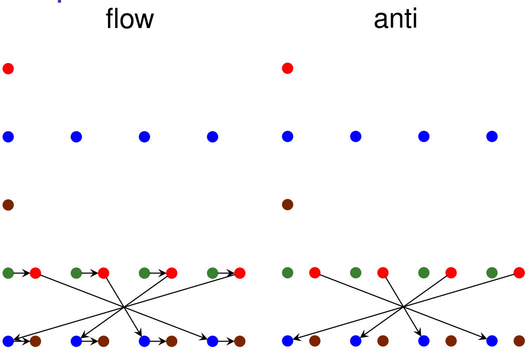

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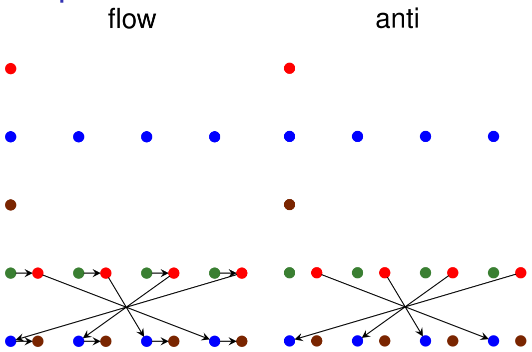

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  - live-in reads
    - $\Rightarrow$ order before all (later) writes
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    - $\Rightarrow$ order after all (earlier) reads
- Output dependences
  - there is a read between the two writes
    - $\Rightarrow$ covered by live-range and anti-dependence
  - the two writes form live-ranges with the same read
    - $\Rightarrow$ preserve order of the writes
  - first write does not appear in a live-range
    - $\Rightarrow$ add output dependence to conditioned validity constraints

Outline

Conclusion

- Enforcing anti-dependences limits scheduling freedom
- Live-range reordering
  - allows anti-dependences to be partly ignored
  - without increasing memory requirements
  - with limited loss of scheduling freedom
- Conditional validity constraints
  - allow live-range reordering during construction of schedule bands
  - available in `PPCG` since version 0.02 (April 2014)
  - crucial for experiments of Baghdadi, Beaugnon, et al. (2015)

Thanks to

- European FP7 project CARP id. 287767
- COPCAMS ARTEMIS project
- Baghdadi, Beaugnon, et al. (2015)

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