Generating SIMD Instructions for Cerebras CS-1 using Polyhedral Compilation Techniques

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Cerebras Systems

January 22, 2020

Outline

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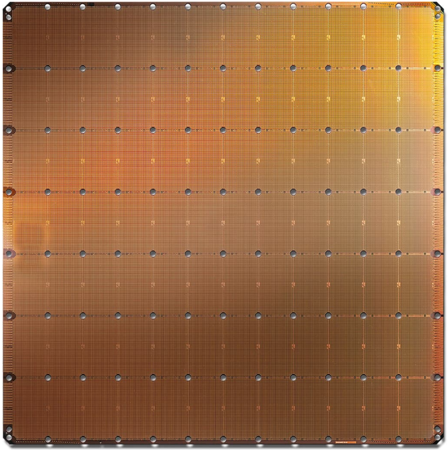
Cerebras CS-1

Largest chip ever built

- $46{,}225\ mm^2$ silicon
- 1.2 trillion transistors
- 400,000 AI optimized cores
- 18 Gigabytes of On-chip Memory
- 9 PByte/s memory bandwidth
- 100 Pbit/s fabric bandwidth
- TSMC 16nm process



Interesting Features

- Dataflow scheduling in hardware
  - Triggered by data
  - Filters out sparse zero data
  - Skips unnecessary processing

Sparse Tensor Communication

Tensor

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0	0	0
57	0	13

Dense Communication

$\xleftarrow{\text{send}}$

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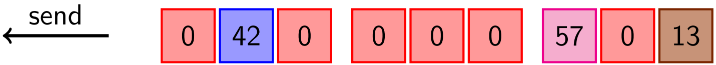

Sparse Communication

- break up tensor into chunks (e.g., rows)
- only send
  - non-zero entry + position in chunk
  - end-of-chunk

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- Dataflow scheduling in hardware
  - Triggered by data
  - Filters out sparse zero data
  - Skips unnecessary processing
- Powerful SIMD Engine
  - Performs *some number* of operations per cycle
  - Mimics normalized loop nest of depth at most four
    $\Rightarrow$ removes overhead of software managed loops

SIMD Instructions

Loop code:

```
handle(uint16_t index, half data) {
  for (int c3 = 0; c3 <= 4; c3 += 1)
    for (int c4 = 0; c4 <= 4; c4 += 1)
      dx_local[2 * dy_index_0 + c3][2 * index + c4] +=
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SIMD instruction:

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void main() {
  configure(/* 5,5; W_local: i,j -> 0,i,j; dx_local: i,j -> i,j */);
  set_base_address(W, &W_local[0][0][0]);
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```

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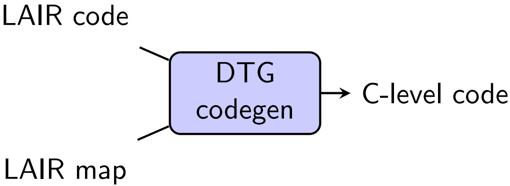
Code Generation Overview



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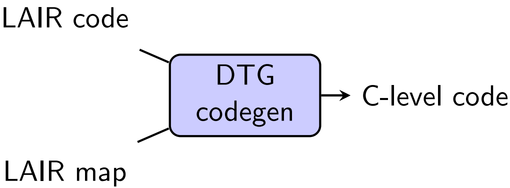

LAIR

⇒ DSL written by hand or extracted from `TensorFlow` (Abadi et al. 2016)

LAIR Example

```
lair matvec<T=float16>(M, N): T W[M][N], T x[N] -> T y[M] {
    all (i, j) in (M, N)
        y[i] += W[i][j] * x[j]
}
```

`lair` node

- defines one or more output tensors in terms of input tensors
- each statement has zero-based rectangular set of instances
- LAIR is single assignment (at tensor level)
- all accesses are affine (not piecewise, not quasi-affine)
- each tensor in a statement is accessed through single index expression

Other nodes combine and/or specialize `lair` nodes
$\Rightarrow$ e.g., $M = 32$ and $N = 16$

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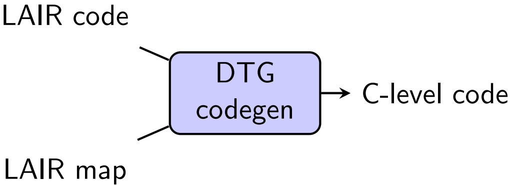

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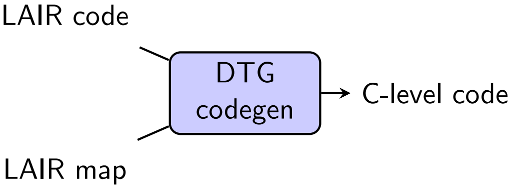

LAIR

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LAIR map contains information in `isl` (V. 2010) notation about

- the size of the target rectangle of PEs
- how input and output tensors are communicated
- where computations are performed

LAIR Map Example

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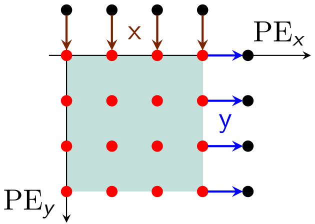
Mapping of 32×16 matrix vector multiplication to 4×4 PEs.




```
size: { PE[4, 4] }
compute_map: { ff[i, j] -> PE[j//4, i//8] }
iport_map: { x[i=0:15] -> [PE[i//4, -1] -> index[i%4]] }
oport_map: { y[i=0:31] -> [PE[4, i//8] -> index[i%8]] }
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Task Graph Construction

Code generation consists of

- Parse LAIR and LAIR map
- Construct task graph
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Task graph construction: split LAIR specification into

- communication tasks
- computation tasks
  Two types:
  - react to incoming tensor element
  - read in entire tensor or operate on local memory

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SIMD Code Generation

$\Rightarrow$ detect sets of computation instances that can be performed by SIMD instructions

$\Rightarrow$ determine

- supported instruction
- "fixed" instance set sizes
- accesses of the form

$$\textit{offset} + \textit{linear in iterators}$$

"fixed" sizes: may depend on PE, but not on tensor element
Otherwise, configuration needs to be performed before each invocation

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Challenge

Recall:

`lair` node guarantees:

- each statement has zero-based rectangular set of instances
- all accesses are affine (not piecewise, not quasi-affine)

SIMD detection requirements:

- "fixed" instance set sizes
- accesses of the form

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Trivial?

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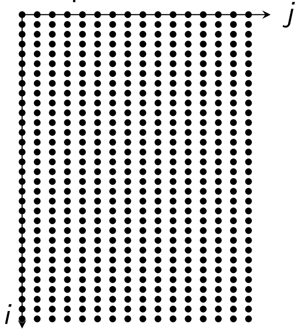

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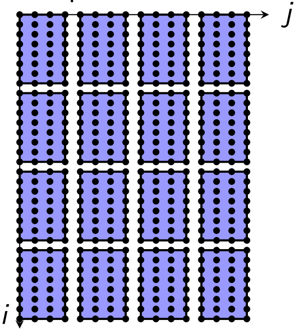

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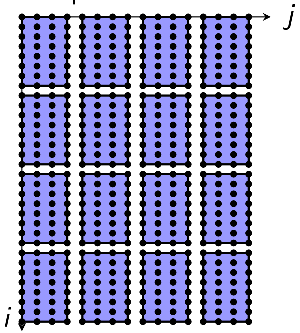

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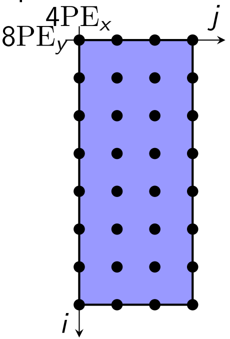

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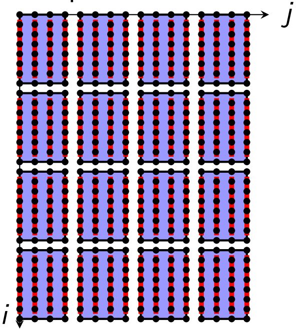

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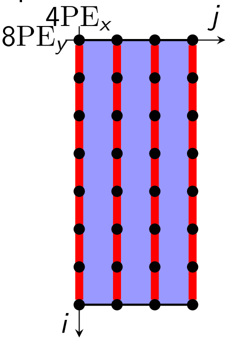

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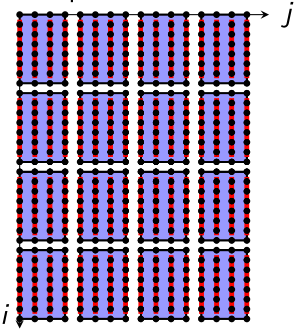

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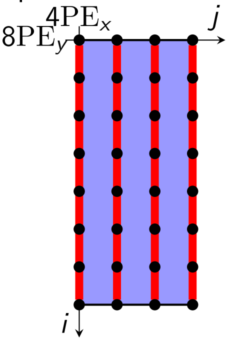

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$\Rightarrow$ Size: $[8, 1]$

$\Rightarrow$ Access to y: $\mathrm{y}[8\mathrm{PE}_y + i']$

(local coordinates: i', j')

Size Computation

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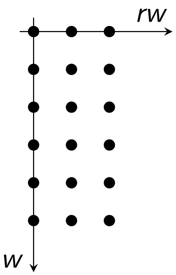

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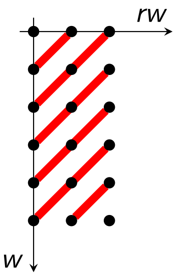

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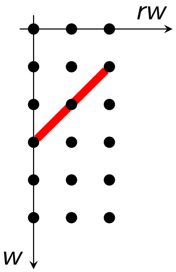

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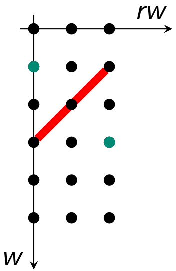

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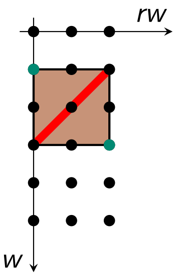

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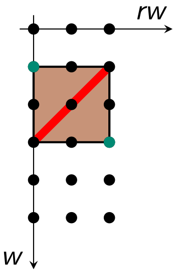

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Variable Compression

Variable compression (Meister 2004):

- pick affine transformation (with inverse) mapping
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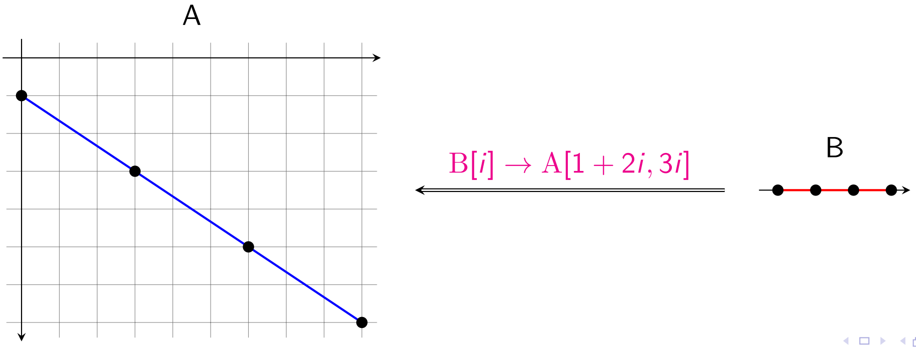

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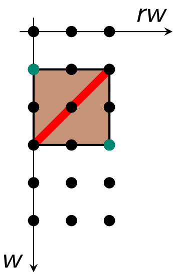

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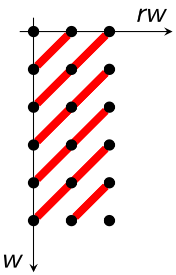

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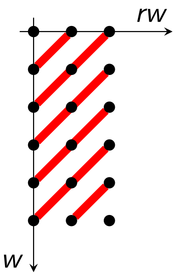

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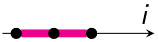

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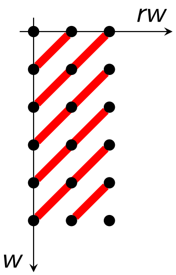

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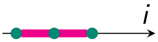

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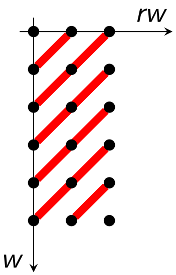

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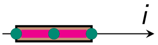

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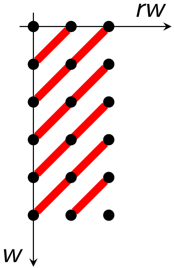
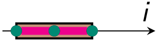
Computation instances:



Compressed instances:



- Arrival of x-value
- Compress
- Compute minimum and maximum
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$\Rightarrow$ a dense box

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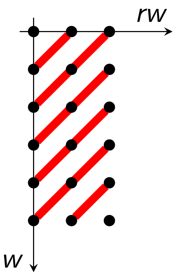

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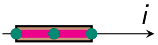

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$\Rightarrow$ a dense box

- Size: $\max - \min + 1$

$\Rightarrow$ [1], [2] or [3] depending on "index"

Fixed Size Box Hull Approximation

Fixed size box hull approximation:

- Result: box containing the input set with
  - variable offset (in particular, may involve "index")
  - fixed size (in particular, does *not* involve "index")
- Approach: look for suitable constraints in representation of input set
- May fail to produce a result

(also used by `PPCG` (V. et al. 2013) to obtain mapping to shared memory)

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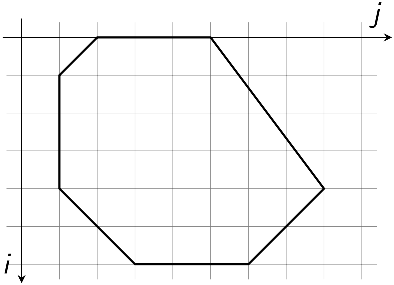

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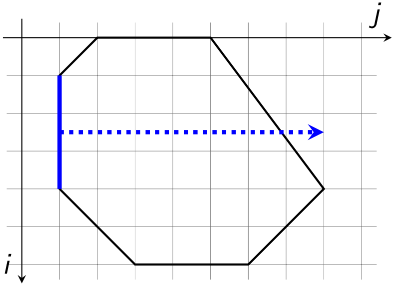

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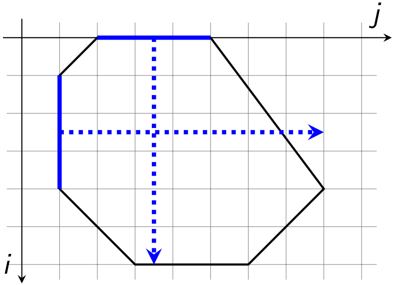

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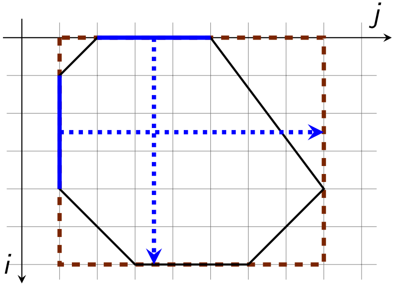

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Size Computation

Input: S: set of instances executed on a PE on arrival of a tensor element

- Apply variable compression to S to obtain S'

- Compute element-wise minimum and maximum of S'
- Construct $\{ \mathbf{x} : \min \leq \mathbf{x} \leq \max \}$
- Check equal to S'
  $\Rightarrow$ S' is a dense box
- Size: $\max - \min + 1$
- Check size does not depend on "index"

Size Computation

Input: S: set of instances executed on a PE on arrival of a tensor element

- Apply variable compression to S to obtain S'
- Try and compute fixed size box hull of S'
  If successful and extra instances write to disjoint locations, then use box size. Stop.
- Compute element-wise minimum and maximum of S'
- Construct $\{\, \mathbf{x} : \min \le \mathbf{x} \le \max \,\}$
- Check equal to S'
  $\Rightarrow S'$ is a dense box
- Size: $\max - \min + 1$
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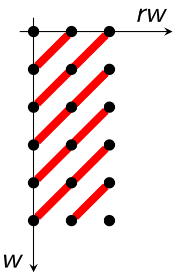

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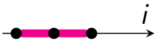

- Arrival of x-value
- Compress

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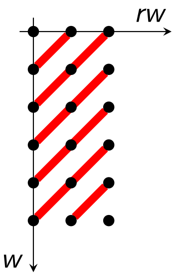

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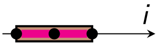

- Arrival of x-value
- Compress
- Try and compute box hull

Convolution

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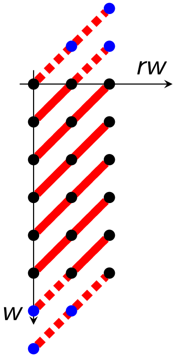

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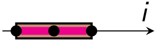

- Arrival of x-value
- Compress
- Try and compute box hull
- Extra instances write to disjoint locations

Outline

Conclusion

- achieving good performance on Cerebras CS-1 requires generation of SIMD instructions
- heuristics based approach can detect opportunities in many cases, using
  - variable compression
  - fixed size box hull approximation
- effective use of polyhedral compilation techniques (other than affine scheduling)

References I

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