Recover Polyhedral Transformations From Polyhedral Schedule

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Polyhedral Representation

2D+1 Representation

Example with polyhedral schedule 2D+1 representation of seidel-2d kernel:

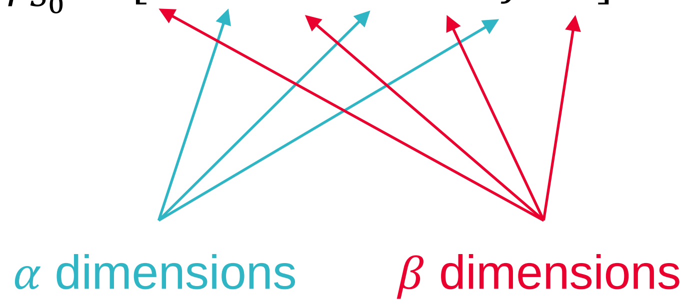
```c
static
void kernel_seidel_2d(int tsteps,
                      int n,
                      DATA_TYPE POLYBENCH_2D(A,N,N,n,n))
{
  int t, i, j;

#pragma scop
  for (t = 0; t <= _PB_TSTEPS - 1; t++)
    for (i = 1; i<= _PB_N - 2; i++)
      for (j = 1; j <= _PB_N - 2; j++)
        A[i][j] = (A[i-1][j-1] + A[i-1][j] + A[i-1][j+1]
                   + A[i][j-1] + A[i][j] + A[i][j+1]
                   + A[i+1][j-1] + A[i+1][j] + A[i+1][j+1])/SCALAR_VAL(9.0);
#pragma endscop

}
```

D corresponds to the number of iterators

$$\phi_{S_0} = [0 \quad t \quad 0 \quad i \quad 0 \quad j \quad 0]$$



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Motivating example

bicg

- Initial schedule

$$\phi_{S_0} = [0 \quad i \quad 0]$$
$$\phi_{S_1} = [1 \quad i \quad 0]$$
$$\phi_{S_2} = [1 \quad i \quad 1 \quad j \quad 0]$$
$$\phi_{S_3} = [1 \quad i \quad 1 \quad j \quad 1]$$

```
#pragma scop
  for (i = 0; i < _PB_M; i++) {
    s[i] = 0;
  }
  for (i = 0; i < _PB_N; i++) {
    q[i] = SCALAR_VAL(0.0);
    for (j = 0; j < _PB_M; j++) {
      s[j] = s[j] + r[i] * A[i][j];
      q[i] = q[i] + A[i][j] * p[j];
    }
  }
#pragma endscop
}
```

Chlore transformation:

reorder([1], [1,0]);
split([1,0], 1);
reorder([], [0,2,1]);
fuse([0]);
reorder([1,0], [1,0]);
split([1,0,0], 2);
split([1,0], 1);
reorder([], [0,2,1]);
fuse([0]);
fuse([0]);
fuse([0,2]);
interchange([0,2,0], 1, 2, 0);
embed([0,1]);
embed([0,0]);

Our recovery transformation:

Split([1, 1], 0);
Fuse([0]);
Fuse([0]);
Fuse([0, 0]);
Fuse([0, 0]);
Interchange([2], 1, 3);

- Compute schedule

$$\phi_{S_0} = [i \quad 0 \quad 0]$$
$$\phi_{S_1} = [i \quad 0 \quad 1]$$
$$\phi_{S_2} = [j \quad i \quad 2]$$
$$\phi_{S_3} = [i \quad j \quad 3]$$

```
#pragma scop
  /* Scattering iterators. */
  int t1, t2;
  int lbp, ubp;

  if ((_PB_M >= 1) && (_PB_M == _PB_N)) {
    lbp=0;
    ubp=_PB_M-1;
#pragma omp parallel for private(t2, )
     for (t1=lbp;t1<=ubp;t1++) {
      s[t1] = 0;
      q[t1] = SCALAR_VAL(0.0);
      for (t2=0;t2<=_PB_M-1;t2++) {
        s[t1] = s[t1] + r[t2] * A[t2][t1];
        q[t1] = q[t1] + A[t1][t2] * p[t2];
      }
    }
  }
#pragma endscop
}
```

*with CLooG, context: _PB_M==_PB_N

L. Bagnères et al, Opening polyhedral compiler's black box, CGO'16

Transformation Primitives

- Transformation primitives either modify the α dimension, either modify the β dimensions.
- All transformation primitives are invertible.

Name	Type	Description
$Reorder(\vec{\beta}, \vec{p})$	β-primitive	Reorder inner-loops or statements directly beneath the given outer-loop.
$Split(\vec{\beta})$	β-primitive	Split outer-loop just before given inner-loop or statement.
$Fuse(\vec{\beta})$	β-primitive	Fuse given loop with the next one on the same depth.
$Embed(\vec{L})$ $Unembed(\vec{L})$	α-primitive	Embed given statements beneath an innermost one-iteration extra loop. Unembed removes the added innermost extra loop.
$Reverse(\vec{L}, d)$	α-primitive	Reverse given output dimension for given statements.
$Grain(\vec{L}, d, c)$ $Densify(\vec{L}, d, c)$	α-primitive	Add some pad between consecutive iterations in given output dimension for given statements. Densify removes some/all of the pad.
$Shift(\vec{L}, d, c, \vec{C})$	α-primitive	Shift given output dimension by some (parametric) coefficient(s) for given statements.
$Interchange(\vec{L}, d_1, d_2)$	α-primitive	Interchange two given output dimensions for given statements.
$Skew(\vec{L}, d_1, d_2, c)$	α-primitive	Skew first output dimension by a coefficient of the second output dimension.
$Reshape(\vec{L}, d, d_{input}, c)$	α-primitive	Skew given output dimension by a coefficient of the given input dimension.

- $\vec{\beta}$: beta-vector targeting an entity
- $\vec{p}$: permutation vector
- $\vec{L}$: list of schedule IDs
- $\vec{C}$: list of parametric shift coefficients
- d, d_1, d_2, d_{input} : an output/input dimension
- c : a scalar coefficient

Recovery Algorithm

Recovery Algorithm

Normalization

$$\begin{aligned}
\phi_{S0} &: [\; 5 \quad i \quad j \quad 1 \quad 0 \quad 0 \quad k \;] \\
\phi_{S1} &: [\; 5 \quad i \quad j \quad 1 \quad 1 \quad 0 \quad k \;] \\
\phi_{S2} &: [\; 5 \quad -i \quad j \quad 1 \quad 1 \quad 0 \quad k \;] \\
\phi_{S3} &: [\; 5 \quad i \quad j \quad 0 \quad 0 \quad 0 \quad k \;] \\
\phi_{S4} &: [\; 5 \quad i \quad 0 \quad 0 \quad 0 \quad 0 \quad k \;]
\end{aligned}$$

(a) Initial

$$\begin{aligned}
&[\; 5 \quad i \quad j \quad 1 \quad k \;] \\
&[\; 5 \quad i \quad j \quad 2 \quad k \;] \\
&[\; 5 \quad -i \quad j \quad 2 \quad k \;] \\
&[\; 5 \quad i \quad j \quad 0 \quad k \;] \\
&[\; 5 \quad i \quad 0 \quad 0 \quad k \;]
\end{aligned}$$

(b) β-collapsing

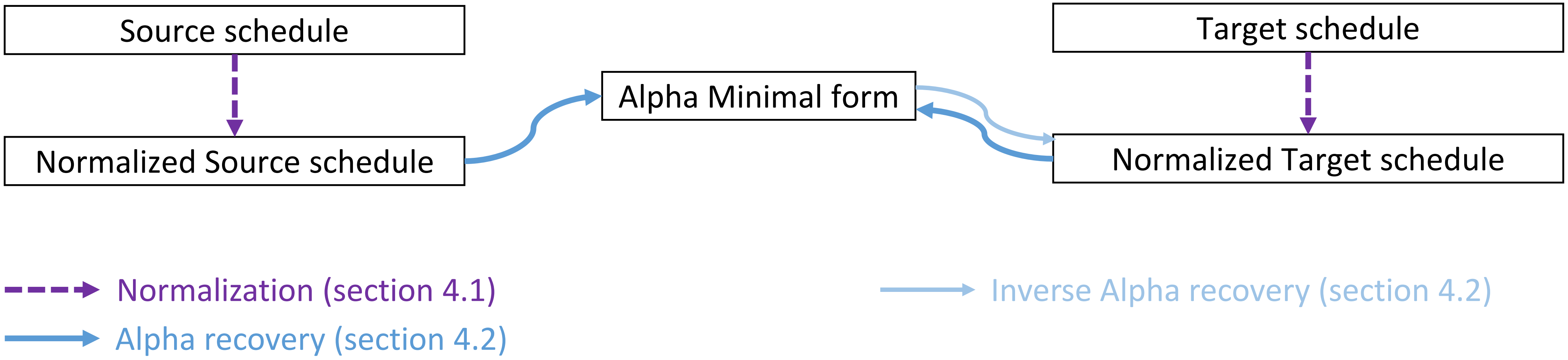
$$\begin{aligned}
&[\; 5 \quad i \quad 0 \quad j \quad 1 \quad k \quad 0 \;] \\
&[\; 5 \quad i \quad 0 \quad j \quad 2 \quad k \quad 0 \;] \\
&[\; 5 \quad -i \quad 0 \quad j \quad 2 \quad k \quad 0 \;] \\
&[\; 5 \quad i \quad 0 \quad j \quad 0 \quad k \quad 0 \;] \\
&[\; 5 \quad i \quad 0 \quad 0 \quad 0 \quad k \quad 0 \;]
\end{aligned}$$

(c) $2d + 1$ format

$$\begin{aligned}
&[\; 0 \quad i \quad 0 \quad j \quad 1 \quad k \quad 0 \;] \\
&[\; 0 \quad i \quad 0 \quad j \quad 2 \quad k \quad 0 \;] \\
&[\; 0 \quad -i \quad 0 \quad j \quad 2 \quad k \quad 1 \;] \\
&[\; 0 \quad i \quad 0 \quad j \quad 0 \quad k \quad 0 \;] \\
&[\; 0 \quad i \quad 0 \quad 0 \quad 0 \quad k \quad 1 \;]
\end{aligned}$$

(d) β-normalization

Recovery Algorithm



Recovery Algorithm

α-recovery

- The α-recovery algorithm search which α transformations are required to achieve Alpha Minimal form
- Alpha Minimal form correspond to the identity between the input domain and the output schedule without the β dimensions:

$$\begin{aligned}
\phi_{S_0} &= [0 \quad i \quad 0 \quad j \quad 1 \quad k \quad 0] \\
\phi_{S_1} &= [0 \quad i \quad 0 \quad j \quad 2 \quad k \quad 0] \\
\phi_{S_2} &= [0 \quad -i \quad 0 \quad j \quad 2 \quad k \quad 1] \\
\phi_{S_3} &= [0 \quad i \quad 0 \quad j \quad 0 \quad k \quad 0] \\
\phi_{S_4} &= [0 \quad i \quad 0 \quad 0 \quad 0 \quad k \quad 1]
\end{aligned}$$

Initial (Normalize) Schedule

$$\begin{aligned}
\phi_{S_0} &= [i \quad j \quad k] \\
\phi_{S_1} &= [i \quad j \quad k] \\
\phi_{S_2} &= [i \quad j \quad k] \\
\phi_{S_3} &= [i \quad j \quad k] \\
\phi_{S_4} &= [i \quad k \quad]
\end{aligned}$$

Alpha Minimal form

- The α transformations are always considers in the same order:
  {Embed, Unembed} – {Densify, Reverse} – Shift – {Skew, Interchange} – Reshape
  - {Embed, Unembed}: modify number of schedule elements/dimensions
  - {Densify, Reverse}: modify the coefficient of an output dimension
  - Shift: add scalar coefficient of an output dimension
  - {Skew, Interchange} - Reshape: reorder of an output dimension, combine output dimensions together
- NB: The normalize source schedule is often already in the Alpha Minimal form

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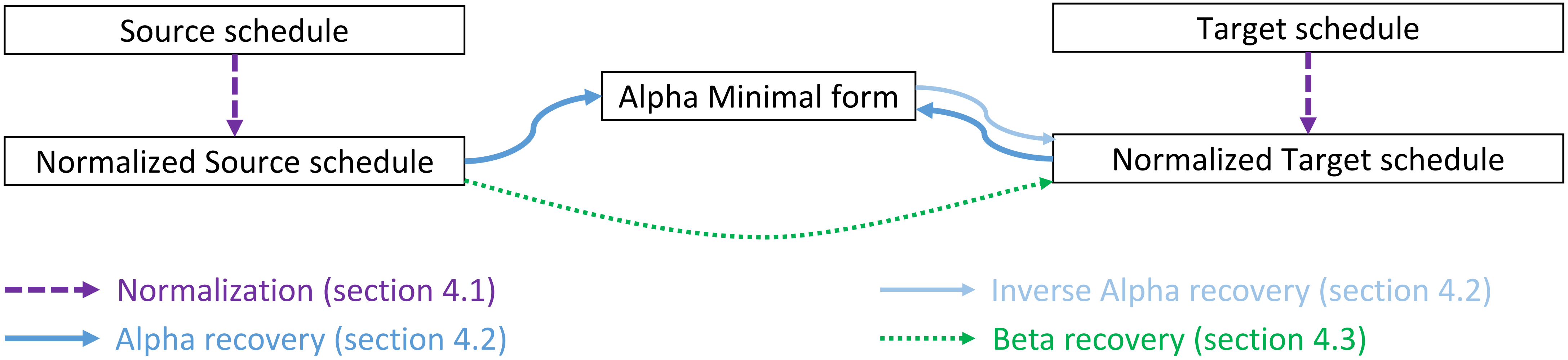
Recovery Algorithm



Recovery Algorithm

β-recovery

- The β-recovery algorithm search which β transformations are required to transform source schedule to target schedule only considering the β dimensions
- The β transformations are repeatly consider in the same order:

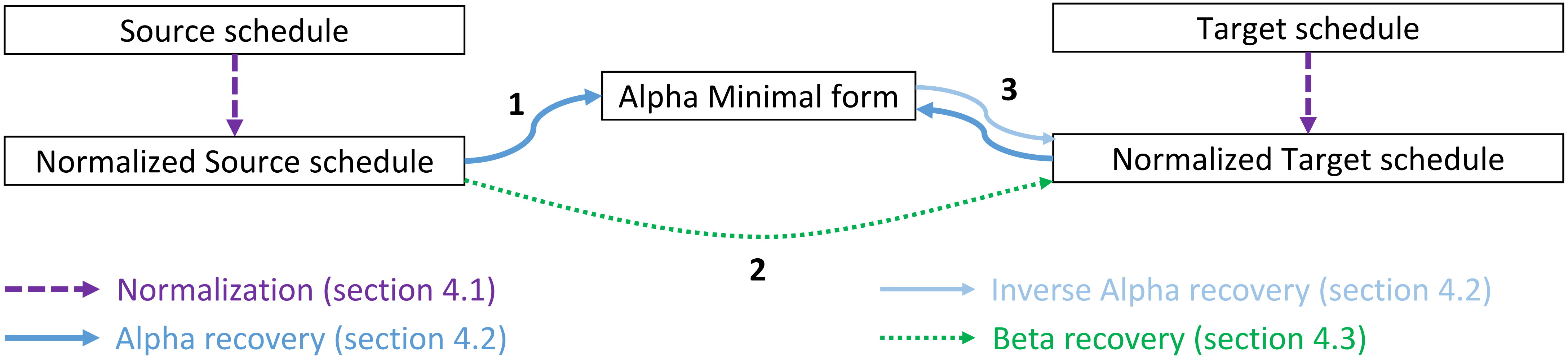
  Reorder – Split – Fuse

  > Reorder: modify the order of the statements on the same depth level

  > Split: separate a statement from the current group of fused statements

  > Fuse: fuse the next statement in the same depth level

Recovery Algorithm



Experiments

seidel-2d (example)

- Source schedule: $\phi_{S_0}^{S} = [0 \quad t \quad 0 \quad i \quad 0 \quad j \quad 0]$
- Target schedule: $\phi_{S_0}^{T} = [t \quad t+i \quad 2*t+i+j]$
- Target Normalized schedule : $\phi_{S_0}^{NT} = [0 \quad \underline{t} \quad 0 \quad \underline{t+i} \quad 0 \quad \underline{2*t+i+j} \quad 0]$

- Chlore (12 transformations)

```
reshape([0,0,0,0], 2, 3, 1);
grain([0,0,0,0], 1, 2);
reshape([0,0,0,0], 1, 3, -1);      find 2*t+i+j
reshape([0,0,0,0], 1, 2, -1);
interchange([0,0,0,0], 2, 3, 0);
skew([0,0,0,0], 2, 3, -1);
reverse([0,0,0,0], 2);
interchange([0,0,0,0], 1, 3, 0);   place 2*t+i+j at right position
skew([0,0,0,0], 1, 3, -1);
densify([0,0,0,0], 1);
interchange([0,0,0,0], 1, 2, 0);   place t+i at right position
skew([0,0,0,0], 1, 2, -1);         find t
```

- Our recovery (3 transformations)

```
Reshape([0], 5, 1, 1);
Reshape([0], 5, 0, 2);
Reshape([0], 3, 0, 1);
```

- Reshape([0], 5, 1, 1): $\phi_{S_0}^{0} = [0 \quad t \quad 0 \quad i \quad 0 \quad i+j \quad 0]$
- Reshape([0], 5, 0, 2): $\phi_{S_0}^{1} = [0 \quad t \quad 0 \quad i \quad 0 \quad 2*t+i+j \quad 0]$
- Reshape([0], 3, 0, 1): $\phi_{S_0}^{2} = [0 \quad t \quad 0 \quad t+i \quad 0 \quad 2*t+i+j \quad 0]$

- Chlore Result schedule:

$$\phi_{S_0}^{chlore} = [0 \quad \phi_{S_0}^{chlore}[4]-i \quad 0 \quad t+i \quad 0 \quad 2*t+i+j \quad 0]$$

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Experiments

Result: Number of transformations recover by Chlore and our tool

- Polybench cases
- Source schedule = initial code
- Target schedule = schedule found with PolyTOPS schedule

Name	Chlore	Our approach
correlation	NA	45
covariance	NA	21
2mm	16	7
3mm	22	13
bicg	14	10
cholesky	13	9
doitgen	1	5
durbin	40	28
gemm	3	3
gemver	7	6
gesummv	7	5
gramschmidt*	29	13
lu	2	4

Name	Chlore	Our approach
mvt	2	2
symm	3	5
syr2k	3	3
syrk	3	3
trisolv	9	5
trmm	3	2
floyd-warshall	0	0
fdtd-2d	41	15
heat-3d	38	12
jacobi-1d	16	4
jacobi-2d	26	8
seidel-2d	12	3

G. Consolaro et al, PolyTOPS: Reconfigurable and Flexible Polyhedral Scheduler, *CGO'24 [Accepted]*

Conclusion and Future Work

- Describe an algorithm to recover transformation primitive that are applied between two polyhedral schedules

- Show that a restricted numbers of primitives is required for recovery

- Show that the set of recover primitive can be heavily reduce comparing with existing tool

- Extend α transformation primitive and α recovery with *stripmine* transformation

- Convert our transformation primitives to primitives for others tools

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