Which is the Best Farkas Multipliers Elimination Method?

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Context

Legality is a key feature in most Polyhedral scheduling algorithms, but :

- Initially, the Legality constraints on the scheduling coefficients are non-linear constraints
- Farkas lemmal[1] is used to linearize the constraints
- New variables "Farkas multipliers" are therefore introduced to define the resulting linear system

[1] Julius Farkas. "Theorie der einfachen Ungleichungen.". In: *Journal für die reine und angewandte Mathematik (Crelles Journal)* 1902.124 (1902), pp. 1–27. DOI: doi:10.1515/crll.1902.124.1.

Motivation

Carrying Farkas multipliers as part of the scheduling problem is problematic and inefficient :

- We do not care about their values
- They increase the number of variables in the linear system considerably:
  - The problem becomes Harder to solver for ILP solvers
  - Longer compilation time
  - Source of errors and risk of reliability

What is the best way to eliminate them ?!

Farkas multipliers elimination methods - FME

The idea of Fourier-Motzkin-Elimination:[2]
Given a system of inequalities with $k+1$ variables, it possible to obtain a system with k variables with no alteration to the solution space (in $\mathbb{R}^k$).

However, our system involves equalities and inequalities.
$\rightarrow$ Many ways to handle the equalities $e_i = 0$:

- *Naive* : $e_i \geq 0 \bigwedge e_i \leq 0$ — $2n$ *new inequalities*
- *Smart*: $e_i \geq 0 \bigwedge \sum e_i \leq 0$ — $n+1$ *new inequalities*
- Leveraging the equalities to pre-eliminate some Farkas multipliers by applying a series of linear combinations. — 0 *new inequalities*

[2] George B. Dantzig and B. Curtis Eaves. "Fourier-Motzkin elimination and its dual". In: *Journal of Combinatorial Theory, Series A* 14.3 (1973), pp. 288–297. ISSN: 0097-3165. DOI: 10.1016/0097-3165(73)90004-6.

Farkas multipliers elimination methods - Cone Projection

Projection using Cones and Chernikova's algorithm:
It is possible to project Farkas multipliers from the system using polyhedra and cone representatios with PolyLib[3]

- The constraints are translated from the Matrix form $AX \geq B|CX = D$ to Cone from using rays and vertices.
- The produced constraints are guaranteed to be minimal
- Chernikova's algorithms[4] is used to assure the minimality ($O(n^3)$ complexity)

[3] Vincent Loechner. *PolyLib: A library for manipulating parameterized polyhedra*. 1999. URL: https://icps.u-strasbg.fr/polylib/ (visited on 2022).

[4] NV Chernikova. "Algorithm for Finding a General Formula for the Non-Negative Solutions of a System of Linear Inequalities". In: *USSR Computational Mathematics and Mathematical Physics* 5.2 (1965), pp. 228–233. DOI: 10.1016/0041-5553(65)90045-5.

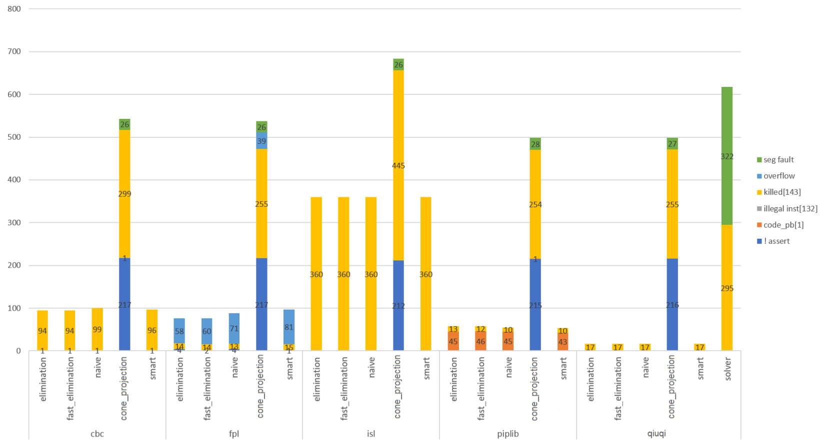
Experimental setup

To determine which Farkas multipliers elimination method is the best:

- We evaluated the 5 Farkas Elimination methods *Naive, Smart, Elimination, Fast_Elimination & ConeProjection* on 7500 Kernels (extracted from Deep Learning models from MindSpore-Akg [2])
- 5 distinct ILP solvers were used to eliminate the solver bias [Piplib [6], Fpl [9], QiuQi, Cbc [7] & isl [10]]
- More than 200 000 executions were performed on a 32 cores Intel Xeon Silver 4215 CPU at 2.50GHZ
- In a Pluto[5] style algorithm

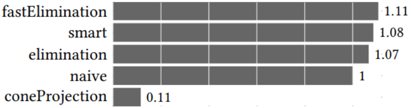
[5] Uday Bondhugula et al. "A practical automatic polyhedral parallelizer and locality optimizer". In: *PLDI '08: Proceedings of the 29th ACM SIGPLAN Conference on Programming Language Design and Implementation*. Tucson, AZ, USA, June 2008, pp. 101–113. ISBN: 978-1-59593-860-2. DOI: 10.1145/1375581.1375595.

Comparing the number of errors

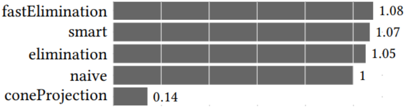


- *Cone_Projection is less reliable across all solvers*
- FME variations are equally the most reliable

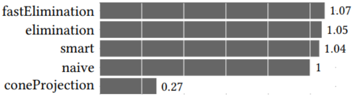
Comparing global scheduling-time



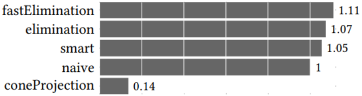
(d) with Piplib solver



(e) with QiuQi solver



(a) with CBC solver

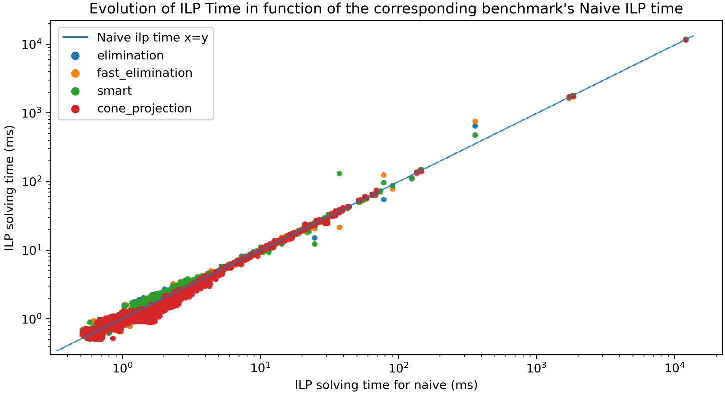


(b) with FPL solver

- *Fast_Elimination* is the fast elimination method just slightly better.
- *Cone_Projection* is x3 to x9 slower than *Naive*.

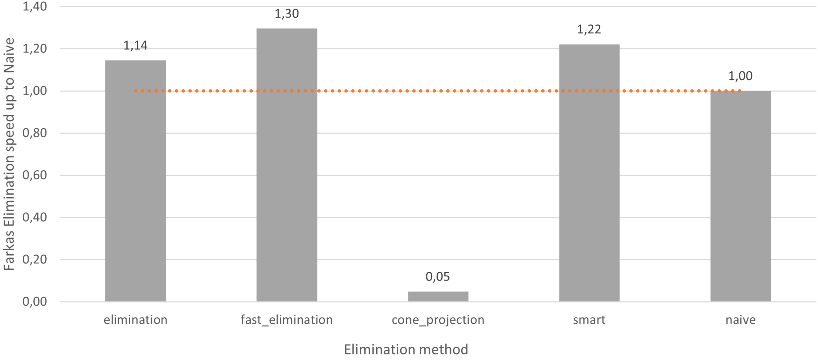
Why and how ?

ILP solving time



- The influence of the number of constraints generated by the different Farkas multipliers elimination methods is limited.

Farkas multipliers Elimination time



- The overhead of *Cone_Projection* makes is x20 slower than other methods
- *Fast_Elimination* > *Smart* > *Elimination* > *Naive* >> *Cone_Projection*

Conclusion

- Pre-elimination of Farkas multipliers using explicit equalities and no *Cost function* is the best strategy for all solvers
- *ConeProjection* improves the ILP resolution time by 14% (because constraints are minimal) but the overhead is too high ($x20$ slower in Farkas Elimination) which makes is unusable in practice
- The Elimination method has very limited impact on ILP solving time (14% improvement with minimal constraints & 2% between FME variants)
- Optimizing Farkas multipliers Elimination method is key to achieve efficient scheduling-time.

Thank You !!
Questions ?

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References

[1] Uday Bondhugula et al. "A practical automatic polyhedral parallelizer and locality optimizer". In: *PLDI '08: Proceedings of the 29th ACM SIGPLAN Conference on Programming Language Design and Implementation*. Tucson, AZ, USA, June 2008, pp. 101–113. ISBN: 978-1-59593-860-2. DOI: 10.1145/1375581.1375595.

[2] L. Chen and Y. Zeng. *Deep Learning and Practice with MindSpore*. Cognitive Intelligence and Robotics. Springer Nature Singapore, 2021. ISBN: 9789811622335.

[3] NV Chernikova. "Algorithm for Finding a General Formula for the Non-Negative Solutions of a System of Linear Inequalities". In: *USSR Computational Mathematics and Mathematical Physics* 5.2 (1965), pp. 228–233. DOI: 10.1016/0041-5553(65)90045-5.

[4] George B. Dantzig and B. Curtis Eaves. "Fourier-Motzkin elimination and its dual". In: *Journal of Combinatorial Theory, Series A* 14.3 (1973), pp. 288–297. ISSN: 0097-3165. DOI: 10.1016/0097-3165(73)90004-6.

[5] Julius Farkas. "Theorie der einfachen Ungleichungen.". In: *Journal für die reine und angewandte Mathematik (Crelles Journal)* 1902.124 (1902), pp. 1–27. DOI: doi:10.1515/crll.1902.124.1.

[6] Paul Feautrier, Jean-François Collard, and Cédric Bastoul. *PIP/PipLib*. 2009. URL: http://www.piplib.org/ (visited on 2022).

[7] John Forrest and Robin Lougee-Heimer. "CBC user guide". In: *Emerging theory, methods, and applications*. INFORMS, 2005, pp. 257–277. ISBN: 1-877640-21-2. DOI: 10.1287/educ.1053.0020.

[8] Vincent Loechner. *PolyLib: A library for manipulating parameterized polyhedra*. 1999. URL: https://icps.u-strasbg.fr/polylib/ (visited on 2022).

[9] Arjun Pitchanathan et al. "FPL: Fast Presburger arithmetic through transprecision". In: *Proceedings of the ACM on Programming Languages* 5.OOPSLA (Oct. 2021-10). ISSN: 2475-1421. DOI: 10.1145/3485539.

[10] Sven Verdoolaege. "*isl*: An integer set library for the polyhedral model". In: *ICMS 2010: Third International Congress on Mathematical Software*. Vol. 6327. Lecture Notes in Computer Science. Kobe, Japan, Sept. 2010, pp. 299–302. DOI: 10.1007/978-3-642-15582-6_49.